AN

APPENDIX

TO THE

ELEMENTS OF EUCLID,

IN SEVEN BOOKS.

CONTAINING

Forty-two moveable Schemes for forming the various Kinds of Solids, and their Sections, by which the Doctrine of Solids in the Eleventh, Twelfth, and Fifteenth Books of Euclid is illuſtrated, and rendered more eaſy to Learners than heretofore.

BOOK

- I. Contains the Five regular Solids.
- II. Shews the Inſcription and Circumſcription thereof, as ſet forth in the Fifteenth Book of the Elements.
- III. Exhibits a great Variety of irregular Solids.
- IV. Contains ſundry Sorts of Priſms.
- V. Various Kinds of Pyramids, and Fruſtrums thereof.
- VI. Some difficult Propoſitions in the Eleventh and Twelfth Books.
- VII. The Cone and its ſeveral Sections.

SECOND EDITION.

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PREFACE.

THE approbation ſhewn to the firſt edition of this work, and the many applications made for it while out of print, are motives that have encouraged me not only to iſſue this ſecond edition, but alſo to make ſundry additions and improvements to it, in which the Solids contained in the firſt and third books hereof, with the addition of their ſection-planes, are ſhewn by a new and more expreſſive method of exhibiting them; the number of irregular ſolids is alſo conſiderably increaſed by the addition of others, whoſe formations, far as I know, are wholly new; and the ſixth book is made more extenſively uſeful by a farther conſideration of the doctrine of Planes deſcribed in the definitions and propoſitions of the eleventh and twelfth books of Euclid, the ſame being more particularly illuſtrated and explained by moveable diagrams having the ſame lines and letters thereon as are in the plano-ſchemes belonging to thoſe books, which being duly elevated, the ſeveral planes and lines heretofore deſcribed in plano appear in the very poſitions they are to be conſidered, or conceived in the mind of the learner, principally adapted to the ſchemes given of thoſe books in the edition by profeſſor Simſon

of

of Glaſgow, together with others ſuited to thoſe in the ſeventh and eighth books of the elements of Geometry by my predeceſſor; but as theſe things would augment the preſent work too much, and that thoſe who have the firſt Edition hereof may have an opportunity of obtaining theſe additions without detriment to their former purchaſe, they are reſerved for a ſecond or ſupplemental volume to what is herein contained.

J. L. C.

APPENDIX to the Elements of EUCLID.

BOOK I.

Of the Five regular Solids.

TO form the Solids deſcribed in this book.

Raiſe up the moveable parts of the ſcheme, and fold them around the part that is ſhaded.

I. *Of the Tetraedron, Plate* 1.

The Tetraedron is a ſolid bounded by four equal equilateral triangles.

II. *Of the Hexaedron, or Cube, Plate* 2.

The Hexaedron, or Cube, is bounded by ſix equal ſquares.

III. *Of the Octoedron, Plate* 3.

The Octoedron is bounded by eight equal equilateral triangles.

IV. *Of the Dodecaedron, Plate* 4.

The Dodecaedron is bounded by twelve equal regular pentagons.

V. *Of the Icoſaedron, Plate* 5.

The Icoſaedron is bounded by twenty equal equilateral triangles.

There are particular rules for finding either the ſuperficial or ſolid content of each of thoſe bodies; but the ſame may be more readily obtained by the following table.

A TABLE,

ſhewing the ſuperficial and ſolid content of each of the five regular ſolids, each ſide * whereof is unity.

Names.	Superficies.	Solidity.
Tetraedron.	1,732051	0,1178511
Hexaedron.	6,000900	1,0000000
Octoedron.	3,464102	0,4714045
Dodecaedron.	20,645729	7,663119
Icoſaedron.	8,660254	2,181695

Uſe of the above table.

To find the ſuperficial content of either of the above-named ſolids, one ſide thereof being given.

Multiply the tabular number that ſtands under *Superficies* in a line with the reſpective ſolid, by the *ſquare* of the given ſide thereof; and

For the ſolid content,

Multiply the tabular number that ſtands accordingly under *Solidity* by the *cube* of the given ſide, the former of thoſe products is the *ſuperficial*, and the latter is the *ſolid* content required.

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* By the word *ſide* is here to be underſtood, the line in which any two ſides of the bounding planes meet each other.

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EXAMPLE

EXAMPLE I.

What is the content ſuperficial and ſolid of a Tetraedron, each ſide being 9 inches?

Superficies.	*Solidity.*
1,732051	0,1178511
81	729
1732051	10606599
13856408	2357022
	8249577
Area 140,296131 Sup.	Area 85,9134519 Solid.

EXAMPLE II.

Suppoſe each ſide of the Hexaedron, or Cube, 9 inches?

Superficies.	*Solidity.*
81	729
6,	1
486,	729

EXAMPLE III.

Suppoſe each ſide of the Octoedron 9 inches?

Superficies.	*Solidity.*
3,464102	,4714
81	729
3464102	42426
27712816	9428
	32998
280,592262	343,6506

EXAMPLE IV.

Suppoſe each ſide of the Dodecaedron 9 inches?

Superficies.	*Solidity.*
20,645729	7,663119
81	729
20645729	689 68071
165165832	1532 6238
	53641 833
1672,304049	5586,413751

EXAMPLE V.

Suppoſe each ſide of the Icoſaedron 9 inches?

Superficies.	*Solidity.*
8,660254	2,181695
81	729
8660254	19635255
69282032	4363390
	15271865
701,480574	1590,455655

BOOK II.

Containing an illuſtration of EUCLID'S *Fifteenth Book.*

Directions for folding the ſchemes together.

PLATE VI.

FIRST form the Tetraedron, by folding together its ſeveral triangles around that particular one which is ſhaded:

Then move its vertex till the other angular point, which is moveable, is ſo much elevated above the plane, that the ſide which connects thoſe two points of the Tetraedron becomes parallel to the plane from which the whole was raiſed.

The Tetraedron being in that poſition, bring the ſeveral parts which conſtitute the Hexaedron around it; thus will the four angular points of the Tetraedron fall exactly in the four angles of the Cube.

PLATE VII.

The Octoedron being firſt formed, fold the Tetraedron over it.

PLATE VIII.

Firſt form the Octoedron, and raiſe it perpendicularly upon one of its angular points, then fold the Cube around it, and the angular points of the Octoedron will then touch the center of each ſide of the Cube.

PLATE IX.

Form the Cube, and incline it a little forward till its ſides biſect thoſe of the Octoedron, when folded around it.

PLATE X.

Form the Dodecaedron, and incline it ſo that its ſides may biſect thoſe of the Icoſaedron, when folded around it.

BOOK III.

Containing ſeveral irregular Solids.

PROBLEM.

TO meaſure an irregular ſolid of any form whatſoever.

General Rule.

Procure a ſuitable veſſel, as a tub, ciſtern, or any ſuch as can be moſt eaſily meaſured; then put the body whoſe content is required into it, and pour into the veſſel ſo much water as will juſt cover the ſolid: mark the ſide of the veſſel where it is even with the ſurface of the water, then take out the ſolid, and obſerve well how far the water deſcends; for that part of the veſſel contained between the higheſt aſcent and loweſt deſcent of the water being meaſured, gives the ſolidity of the body required.

Example. Suppoſe the ſolidity of a very irregular piece of ſtone was required, and that having put it into a ſquare ciſtern, whoſe dimenſions are 50 inches, after having juſt covered it with water, on taking it out I find the water to deſcend 8 inches, from thence to find its content.

50 inches
50

Superficial area 2500 of the baſe of the ciſtern,
8 its depth, or deſcent of the water,

Solidity 20000 of that part of the ciſtern poſſeſſed by the body, and therefore the content of the body required in inches, which, divided by 1728, gives 11,516 feet, &c. the content in ſolid feet.

N. B. To put the water into the veſſel firſt, and then immerſe the ſolid therein, may, in ſome caſes, be a more preferable way of proceeding.

To fold any of the figures contained in this book.

Bring the contiguous parts of each figure around that which is ſhaded, the reſt will then join together, and form the ſolid required.

BOOK IV.

Containing various ſorts of Priſms.

DEFINITION.

A Prism is a ſolid figure comprehended by planes, among which two oppoſite ones are parallel, equal, and ſimilar.

REMARKS.

I. The ſides of all priſms are parallelograms; but the ends or baſes are of various forms; hence it is that priſms receive different names, and are accordingly called triangular, quadrangular, pentangular, hexangular, &c.

II. When a priſm is terminated at its ends by parallelograms, it is then moſt generally termed a parallelopipedon.

III. The method of folding together the ſchemes contained in this book is ſo obvious, that nothing more need here be obſerved concerning it, than only to bring the contiguous parts of each figure around that particular one which is ſhaded.

PROBLEM.

To meaſure any ſort of priſm.

General Rule.

Meaſure from the center of its baſe or end, to the middle of any one of its ſides, multiply that length by half the ſum of all the ſides by which the priſm is bounded, and that product is the area of the baſe; which being multiplied by the whole length of the priſm, gives the ſolid content.

Or thus:

Square the ſide of the baſe, and multiply it by the affixed number in the following table belonging to the figure which the baſe of the priſm is of, and that product gives the area of the baſe; which being multiplied by the length of the priſm gives the content as before.

A GENERAL

A GENERAL TABLE

For finding the area of regular polygons.

Sides.	Names.	Multipliers.
3	Triangle.	,433013
4	Square.	1,000000
5	Pentagon.	1,720477
6	Hexagon.	2,598076
7	Heptagon.	3,633959
8	Octagon.	4,828427

N. B. The parallelopipedon is beſt meaſured by multiplying its length, breadth, and depth into each other.

I forbear troubling the reader here with any particular examples, there being ſufficient in ſuch authors as have wrote on the ſubject of menſuration; for to exhibit the bodies themſelves is the only deſign in which I am now engaged.

BOOK V.

Containing various ſorts of Pyramids, and fruſtrums thereof.

DEFINITIONS.

I. A PYRAMID is a ſolid having a polygon for its baſe, and comprehended under triangles, all meeting together in one point; which point is called its vertex.

II. The fruſtrum of a pyramid is that part which remains, when the top part is cut away by a plane or ſection paſſing through all its ſides parallel to its baſe.

To fold any of the figures contained in this book.

The parts which are ſhaded are ſuch evident indications, as render further directions ſuperfluous.

PROBLEM I.

To meaſure a pyramid.

GENERAL RULE.

Multiply the area of its baſe by a third part of its perpendicular altitude, that product is the ſolid content.

N. B. The perpendicular altitude is meaſured by a line drawn from the vertex to the center of its baſe.

PROBLEM II.

To meaſure the fruſtrum of a pyramid.

RULE.

Multiply the areas of the bottom and top baſes together, extract the ſquare root of that product, and add thereto the ſum of both areas, that total multiplied by one third of the fruſtrum's altitude, gives the ſolidity required.

BOOK VI.

Containing an illuſtration of ſome Theorems in EUCLID'*s Eleventh and Twelfth Books.*

EUCL. XI. *Prop.* xxviii.

THEOREM.

A PLANE paſſing through the diameters of oppoſite planes of a parallelopipedon divides it into two equal priſms. See this illuſtrated in Plates XXXIV. and XXXV.

Directions for folding the ſchemes contrived for illuſtrating the above theorem.

PLATE XXXIV.

I. Raiſe up one half of the figure, and fold it ſo that the point E may fall upon A, and the point F upon B, and raiſe up the parallelogram at the end; thus have you one of the priſms.

II. Then bring over the other part of the ſcheme, ſo that the corner C may likewiſe fall upon A, and the corner D upon B; thus will there be formed the whole parallelopipedon and its ſection made by the plane paſſing through its diagonal A B.

PLATE XXXV.

I. Lift up the figure, and bring the point A to C, and the point B to D.

II. Fold back the reſt of the figure upon the line A B, which is properly cut for that purpoſe; ſo that the two parallelograms which are ſeparated by A B may lie cloſe together.

III. Then bring over the remaining part of the ſcheme, ſo that the points E and F may coincide with C and D; then folding together the four triangles, the whole parallelopipedon will be formed, and alſo its ſection made according to a plane paſſing through the diagonal of its baſe or end.

Eucl. XII. *Prop.* iii.

THEOREM.

A Triangular pyramid may be divided into two equal triangular pyramids, having each the ſame baſe and altitude, and into two equal triangular priſms, which two priſms are together greater than the half of the whole pyramid.

Directions for folding together the ſcheme which explains this propoſition.

PLATE XXXVI.

I. Firſt form the triangular pyramid A C B, by making the line B C coincide with A C.

II. Fold back upon the line B C the triangle numbered 4, ſo that it may lie cloſe to that which is marked 3.

III. Then fold over the ſides 5, 6, 7, bringing the line D E ſo that D may fall upon A, and E upon C, which, with the quadrilateral baſe that is ſhaded, forms one of the priſms.

IV. Then fold back the other part of the figure, ſo that the ſide 8 may be cloſe to that marked 7; thus will there remain a triangular vacuity at the baſe of the pyramid A C B, which muſt be filled by folding together the ſides marked 9 and 10, making the point F fall upon A, and the point G upon C; thus will the other priſm be formed.

V. This done, there remains only to form the other pyramid G I H, which is effected by folding its parts back upon the line G K, and folding round the triangles marked 12, 13, 14; the partition lines whereof are all cut on the front ſide of the figure, which, being folded accordingly, completes the whole pyramid, and exhibits a view of the ſeveral ſections deſcribed in the above theorem.

Eucl. XII. *Prop.* vii.

THEOREM.

EVERY pyramid is the third part of a priſm, having the ſame baſe and height. See this illuſtrated in a triangular priſm by

PLATE XXXVII.

To fold the figure contained in this plate.

I. Firſt form the pyramid ABC, by bringing the point C to A, ſo that the line BC may fall upon AB.

II. Then form the pyramid BAD, by bringing D to B, making AD coincide with AB, and raiſe up the triangle at B.

III. Then turn back the reſt of the figure upon the line AD, ſo that the triangle marked 7 may cloſely adhere to that marked 6.

IV. Laſtly, The vacuity that is now between the two pyramids ſo formed will be filled exactly by folding together the pyramid contained under the triangles marked 8, 9, 10, in ſuch ſort that E be connected with A, and F with B; thus is the whole priſm completed, and the ſections above deſcribed clearly ſeen.

N. B. When the laſt-mentioned pyramid is introduced into the ſpace remaining between the other two, by preſſing a little upon that part which is the upper edge or ſide of the priſm, the formation of the priſm will be rendered more perfect.

BOOK VII.

Containing the Conic Sections.

DEFINITIONS.

I. A Cone is a round ſolid, having a circle for its baſe, and may be conceived to be generated by the revolution of a right-angled triangle turning round on one of the ſides, which includes the right-angle.

II. The axis of the cone is that fixed right line, around which the triangle is ſuppoſed to revolve.

III. The extreme end or point, by which the cone is terminated, is called its vertex.

IV. The fruſtrum of a cone is the part remaining, when its top is cut off by a plane or ſection paſſing through it parallel to its baſe.

REMARKS.

I. The cone and its fruſtrum are meaſured by the ſame rules as obtain for pyramids and their fruſtrums.

II. As a pyramid is the third part of a priſm, having the ſame baſe and altitude, ſo in like manner a cone is the third part of a cylinder, having the ſame baſe and altitude; but the roundneſs of thoſe figures does not admit of an explanation of this truth by this mechanical method of repreſenting them.

Directions for folding together the ſchemes contained in this book.

PLATE XXXVIII.

I. Bend the arc A B evenly round the arc E F, thereby cauſing A to come to E, and B to F.

II. Turn about the triangle B C D upon the ſide B C, ſo as to make B D coincide with the diameter of the baſe E F, and A C to coincide with D C; thus will one half of the cone be formed.

III. In like manner form the other half, by bringing G to E, and H to F, the line G K even upon E F, and the ſides H I and K I even to each other; ſo will the whole cone and the ſection, through its axis, both appear.

PLATE

PLATE XXXIX.

I. Fold the arc A D round the baſe, ſo that A may come to B, and the arc D C in like manner, making C to unite with A at the point B.

II. Make the lines A F and C G unite together, which will form the fruſtrum of a cone cut parallel to its baſe, and ſhew the ſection to be a circle.

PLATE XL.

Fold the arcs A B and B C round the arcs B D E, B F E, and make the ſides A G and C H coincide every where together upwards from the point E, which will repreſent a cone cut oblique to its baſe, and ſhew the ſection to be an ellipſis.

PLATE XLI.

I. Form the whole part of the cone A C B, by bringing the points A and B together, and bending it ſo as to become round as near as may be.

II. Bend regularly the other part of the figure, ſo that the arcs D E, F E, may ſurround the arcs F G, F H.

III. Raiſe up the parabola G I H, making the point I meet the coincident points A and B; thus may you ſee the ſection made by cutting a cone parallel to one of its ſides.

PLATE XLII.

The laſt directions obtain here; for by bringing E and F together, and rounding the part E D F, as was there deſcribed, and the points A and B being brought to G and H, if the hyperbola G I H be then raiſed up, as before directed, the ſection of a cone, cut parallel to its axis, will be thereby ſubjugated to view.

FINIS.

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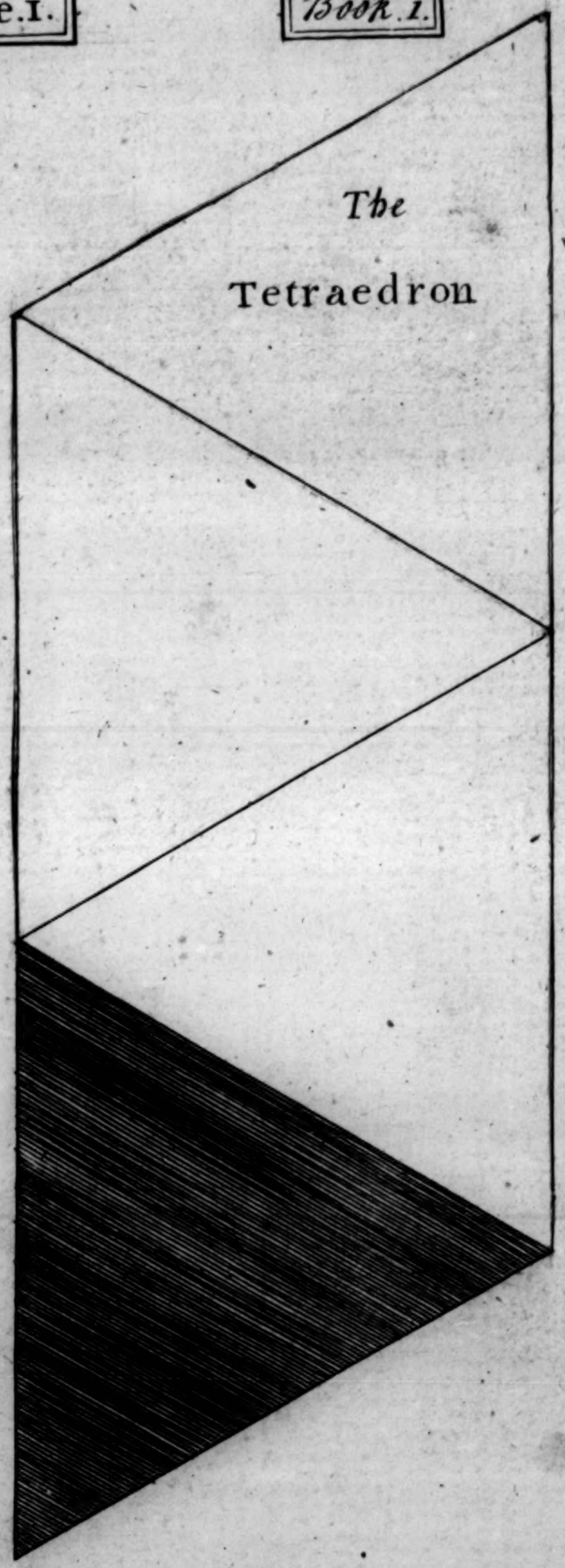
Lately Publiſhed, by the ſame Author,

THE THEORY OF PERSPECTIVE,

Demonſtrated and illuſtrated by moveable SCHEMES,
Which ſhew the ſeveral Planes, Lines, and Points uſed in that Art in the true Poſitions in which they are to be conſidered.

Plate.I.

Book.1.



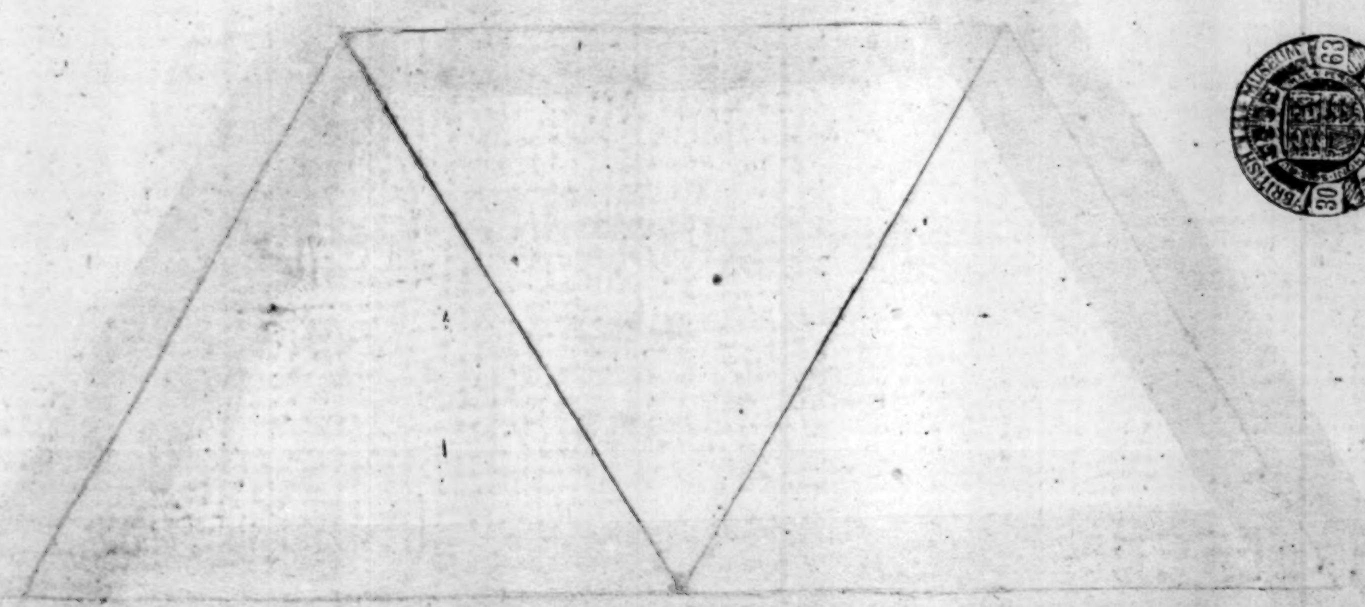
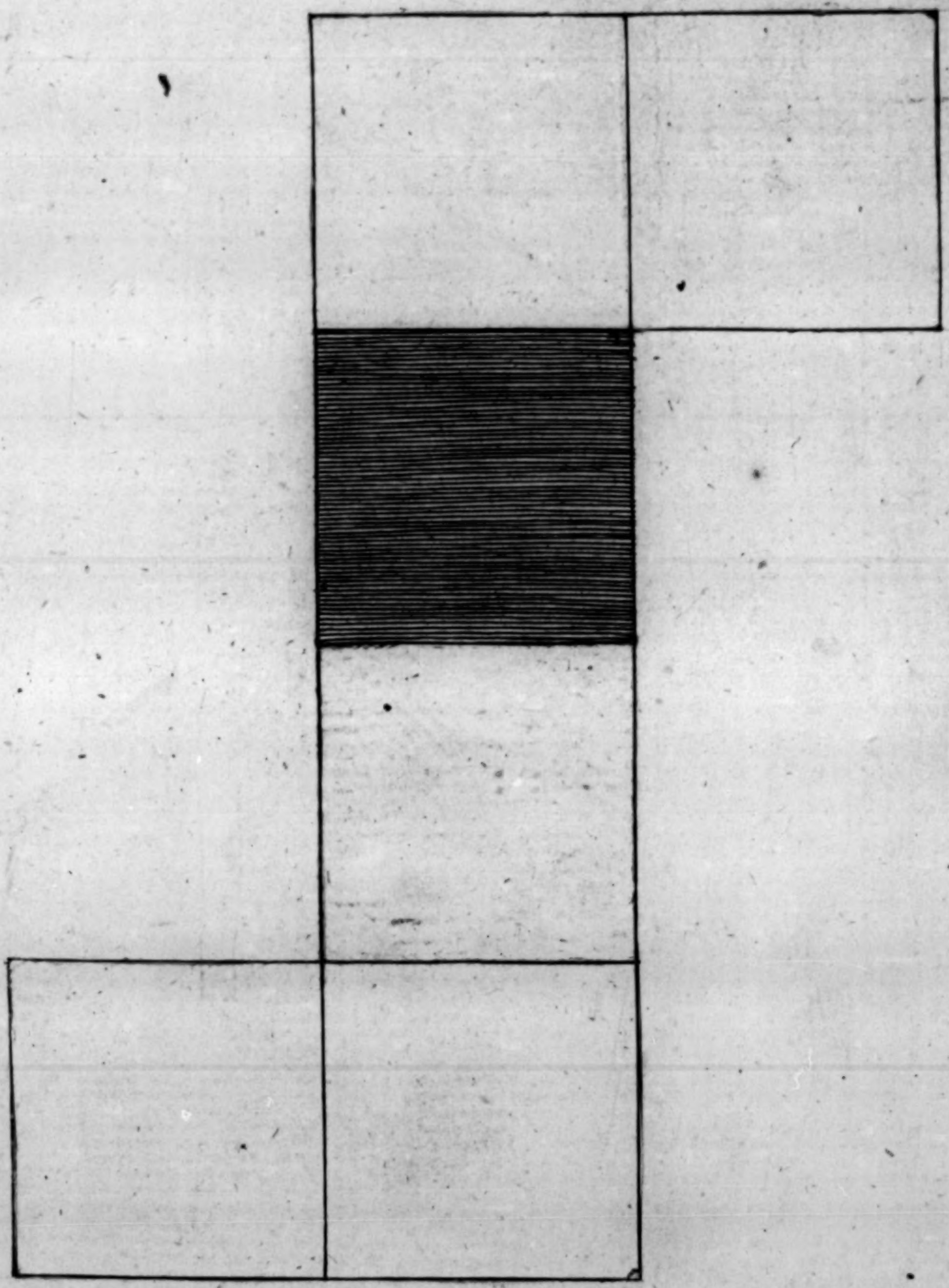
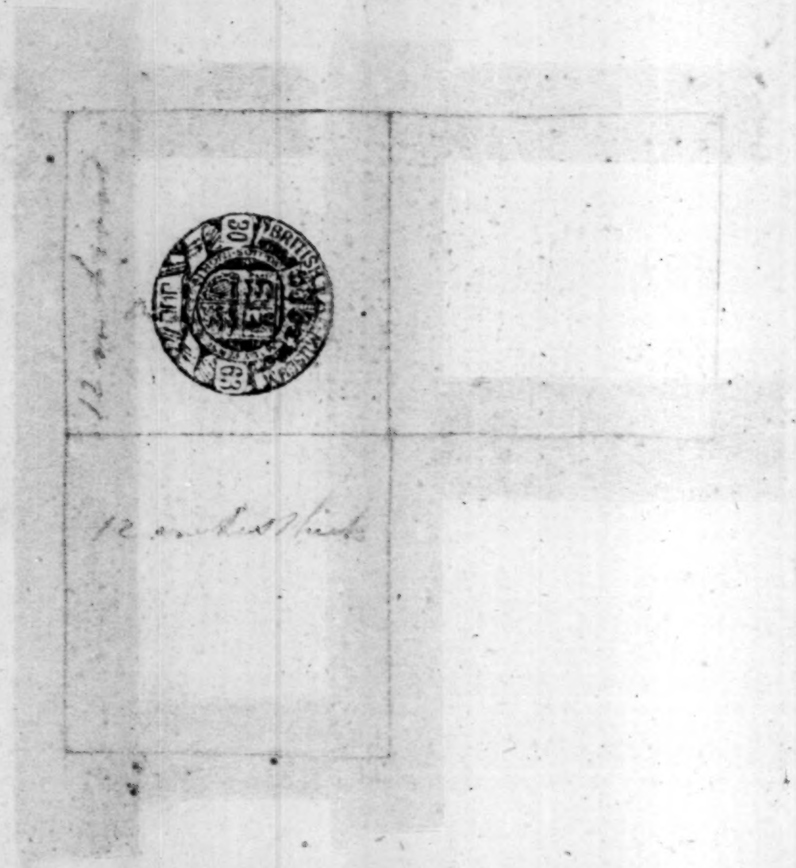


Plate.II.

Book.1.

The Hexaedron *or* Cube.




12 in broad
12 inches thick

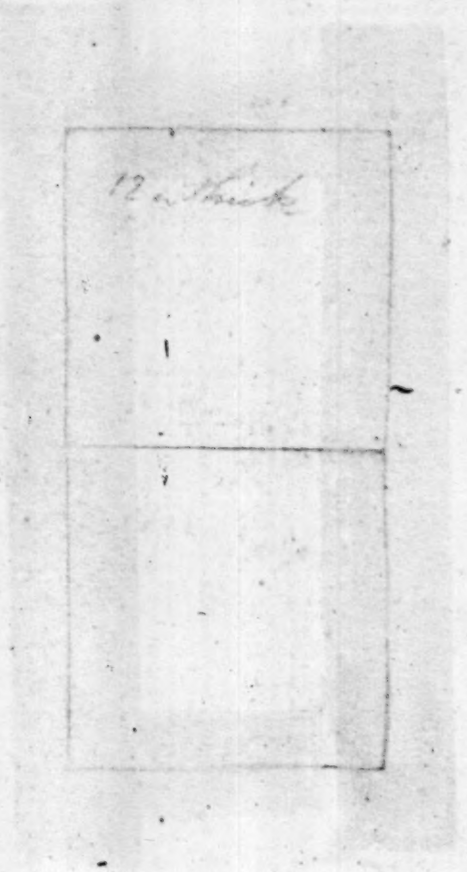

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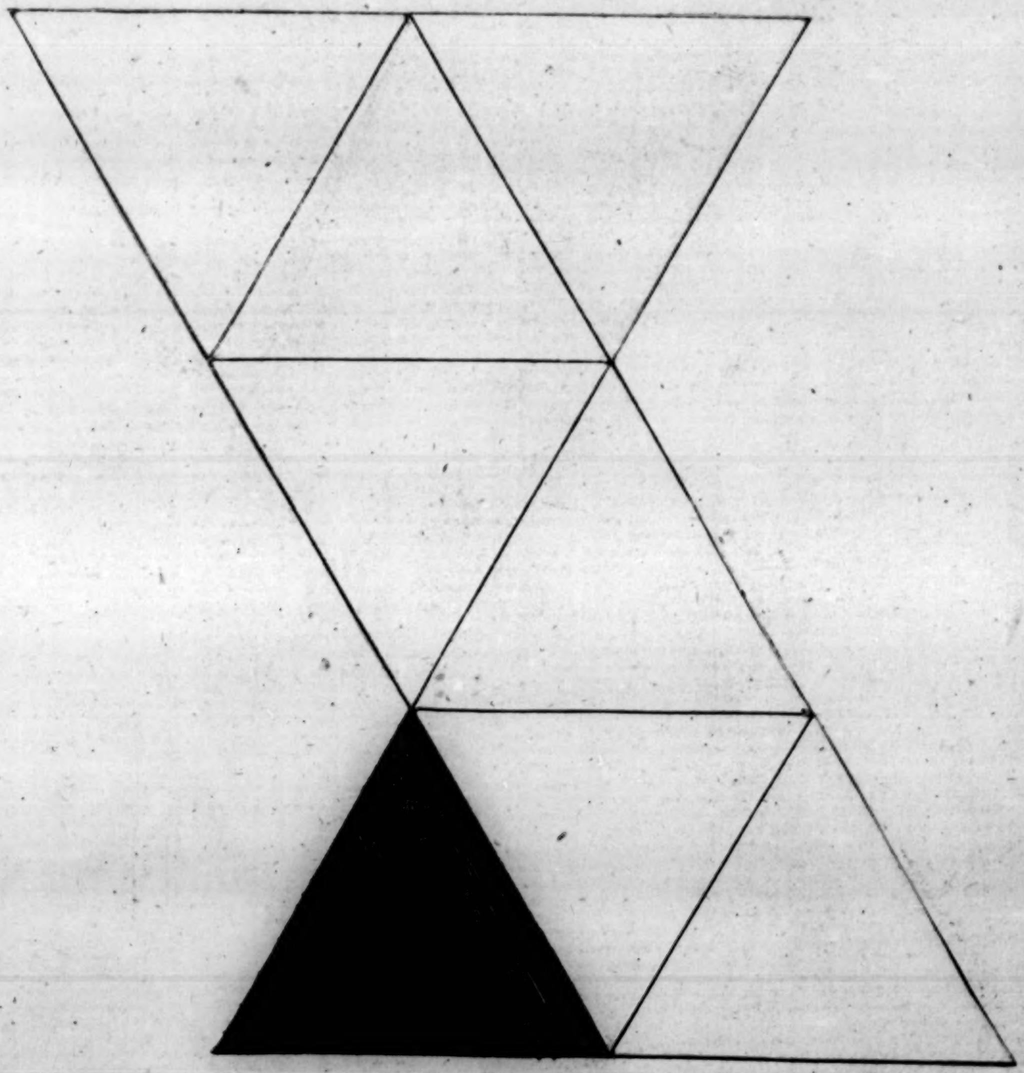
Plate. III.

Book. 1.

The

Octoedron



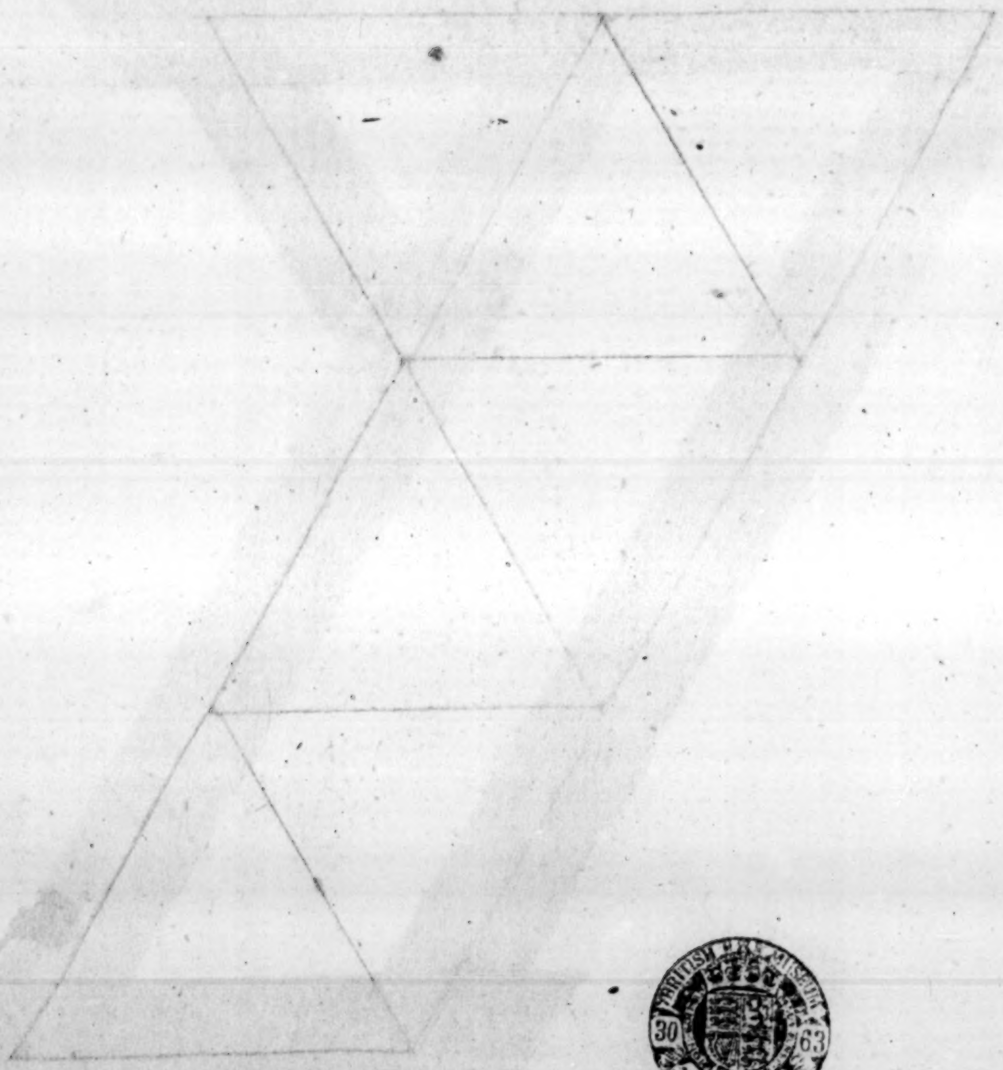


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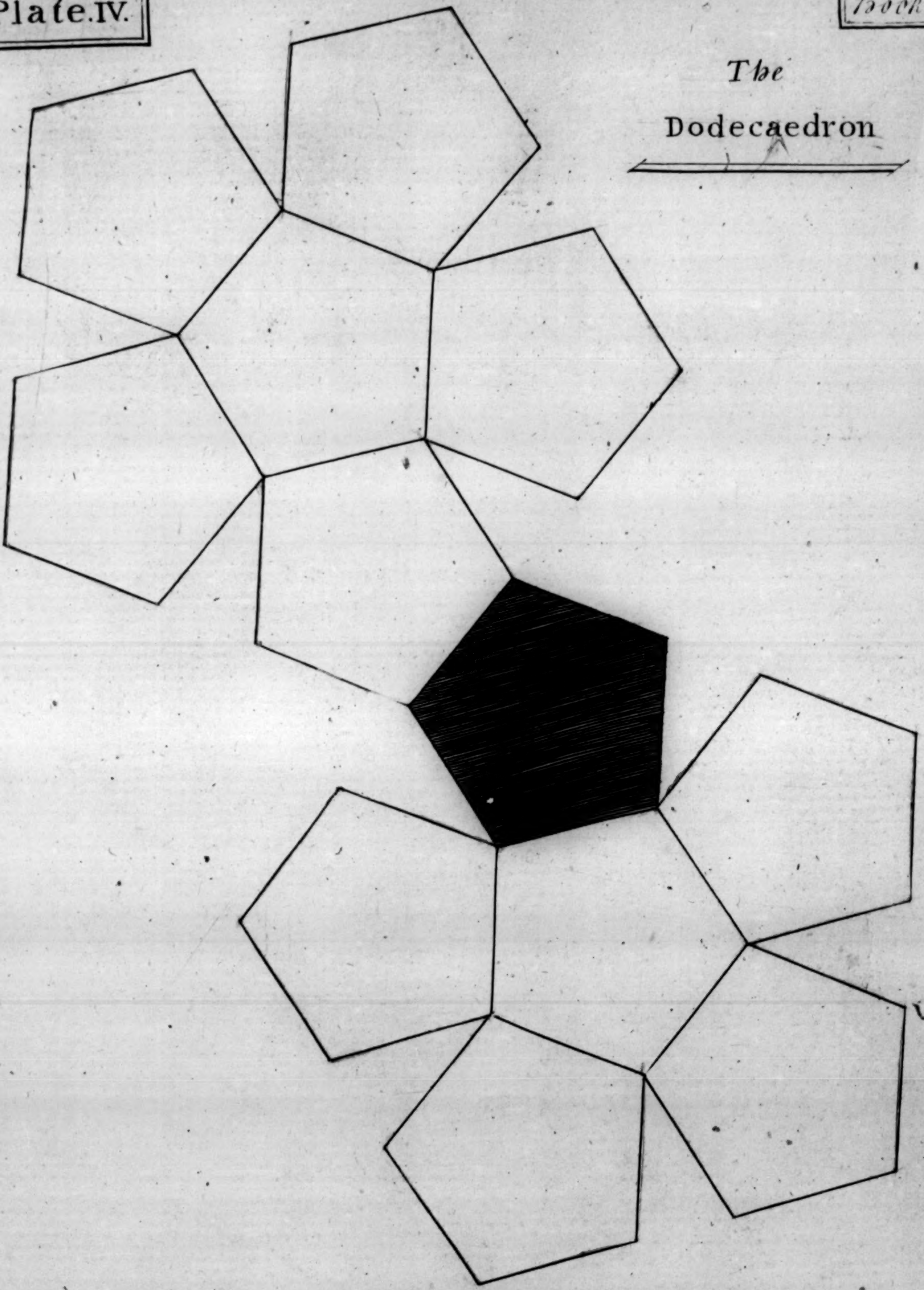
Plate.IV.

Book.1.

The Dodecaedron



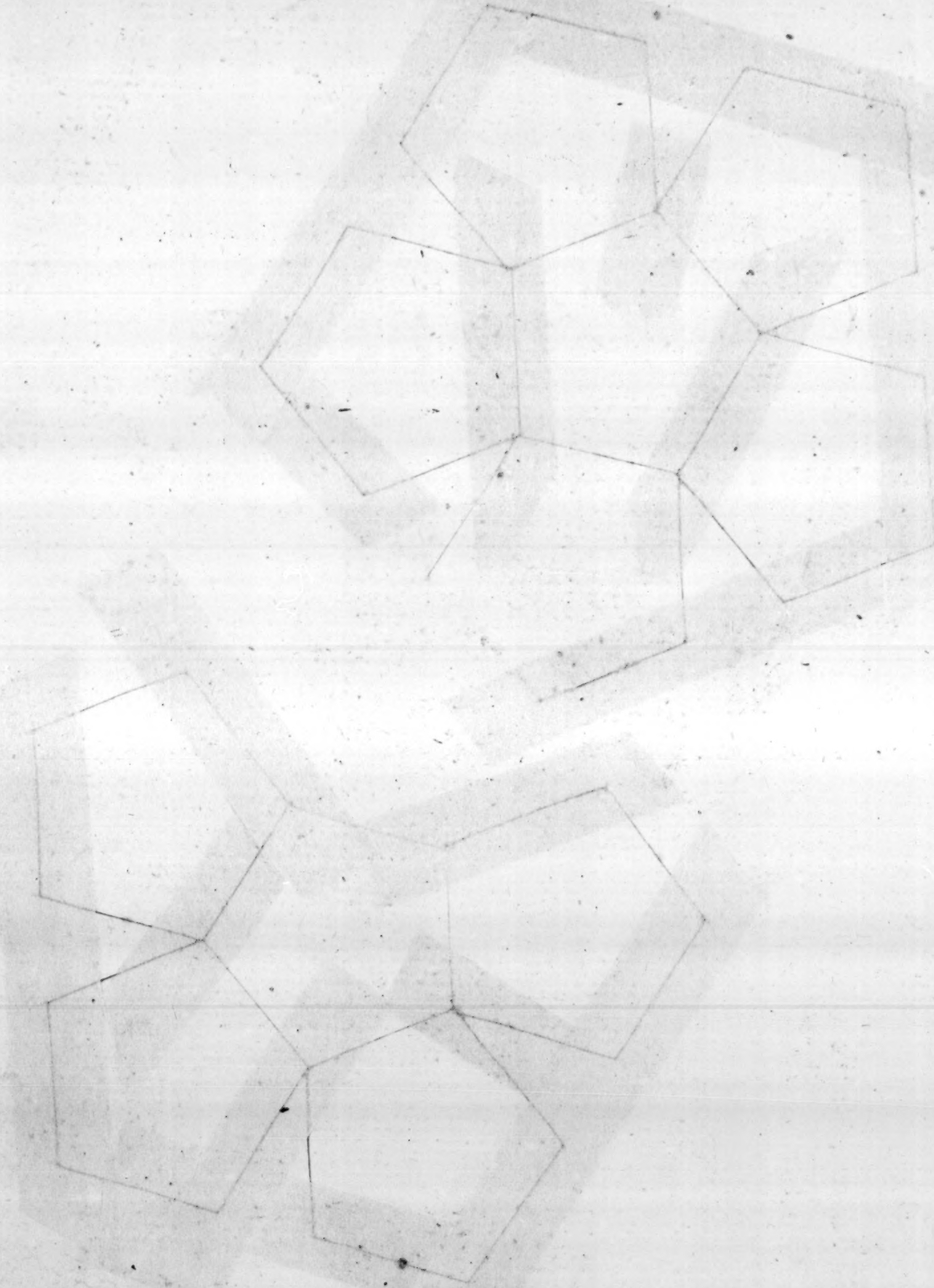
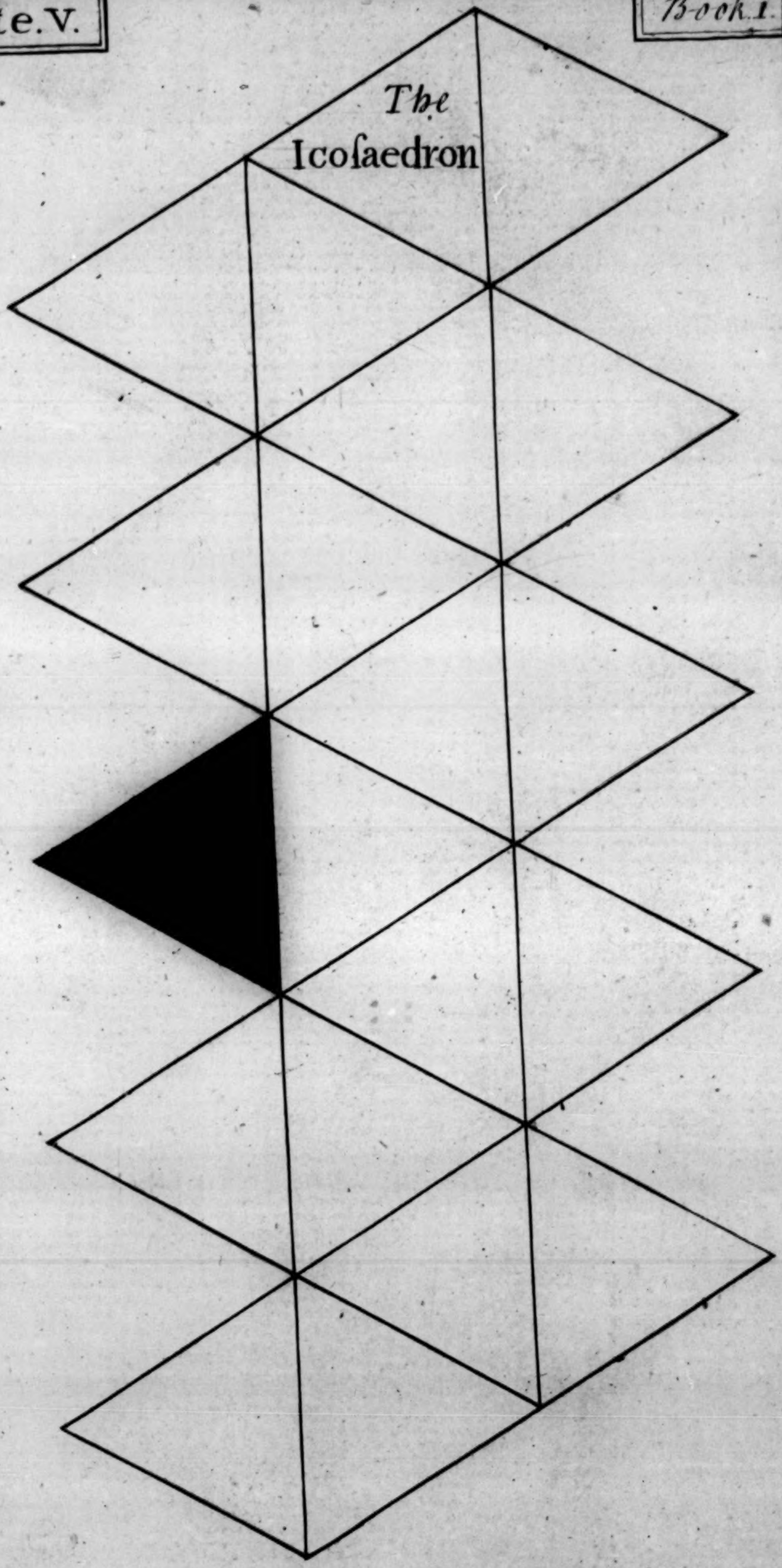


Plate. V.

Book. 1.



The Icoſaedron

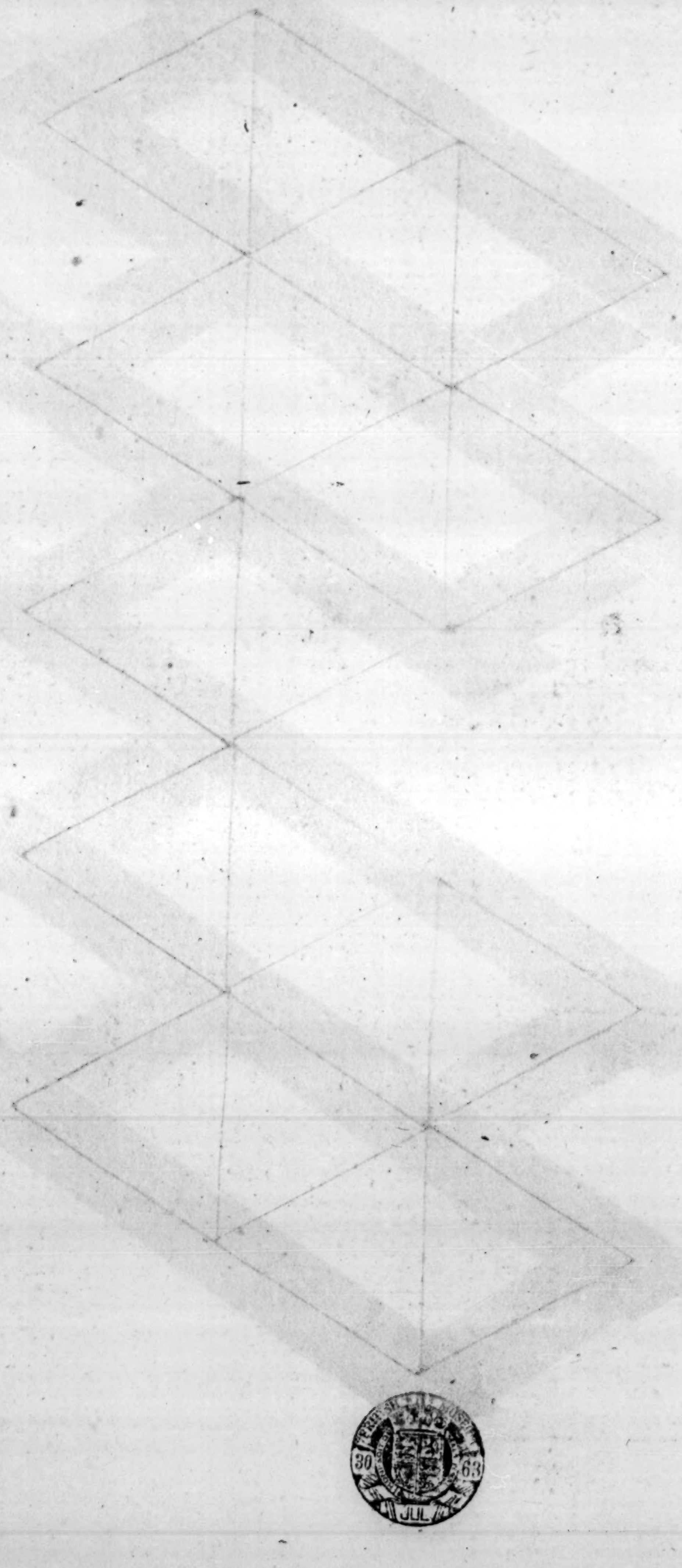


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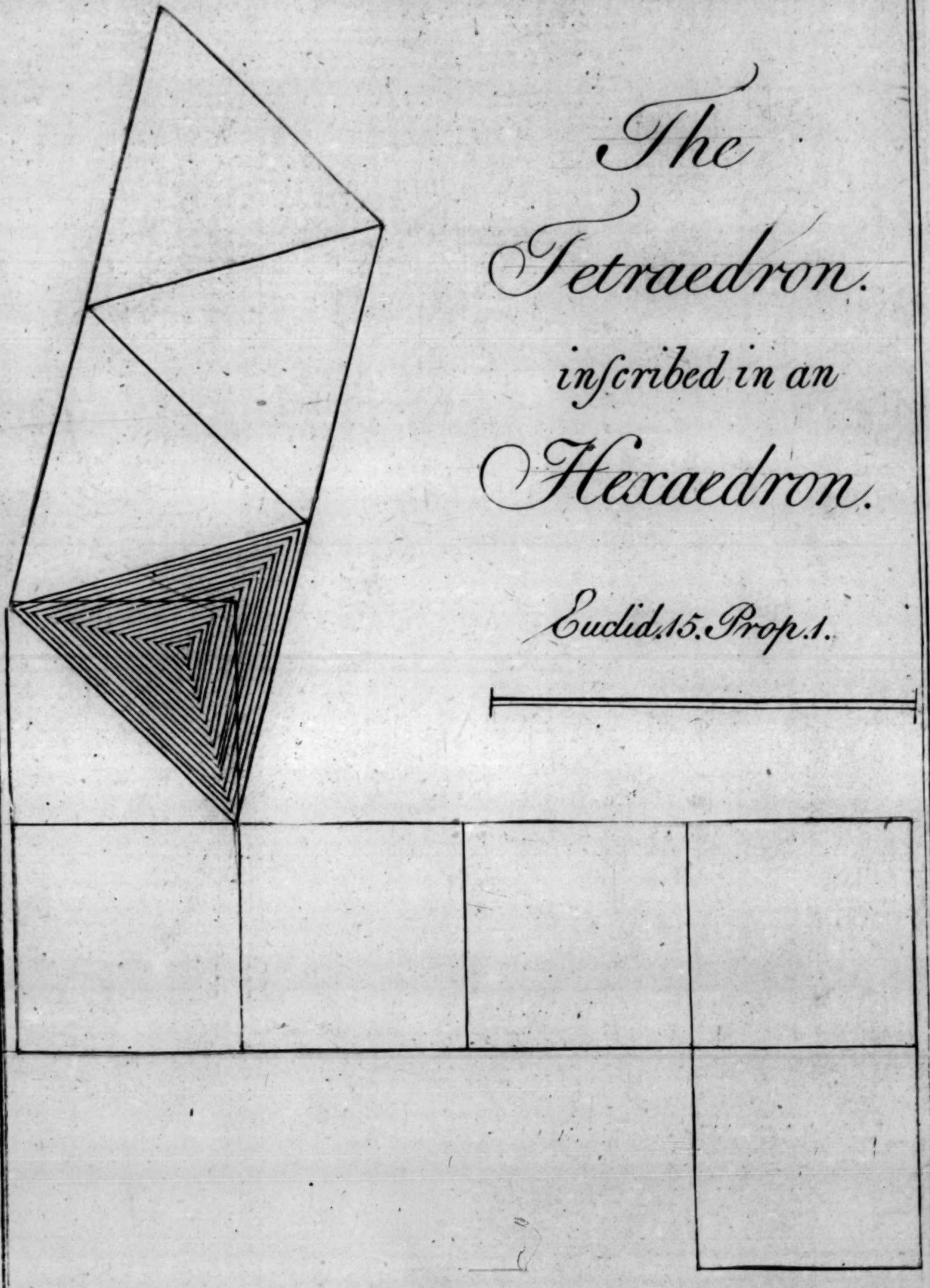
Plate VI.

Book. 2.

The Tetraedron. inſcribed in an Hexaedron.

Euclid.15. Prop.1.



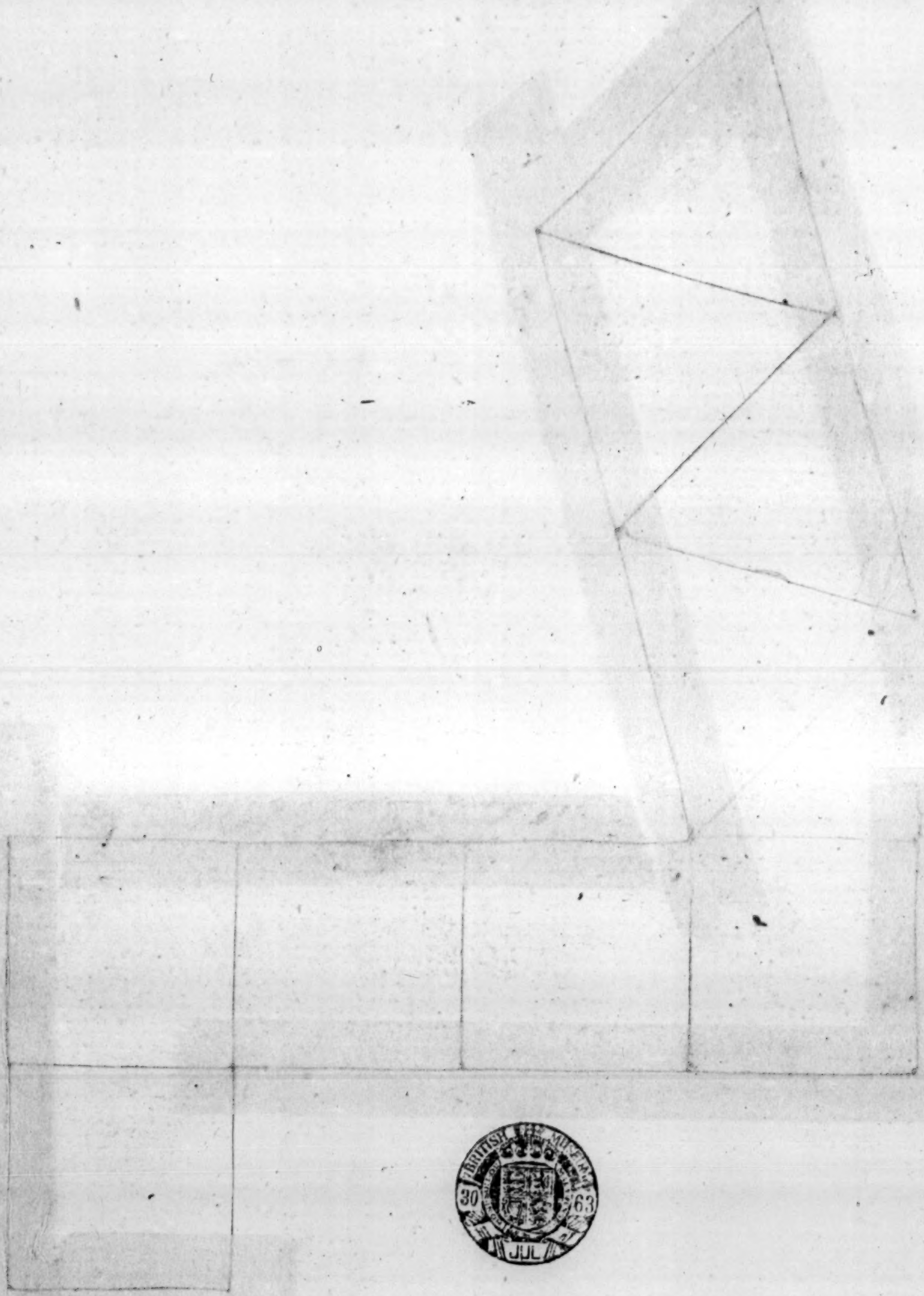
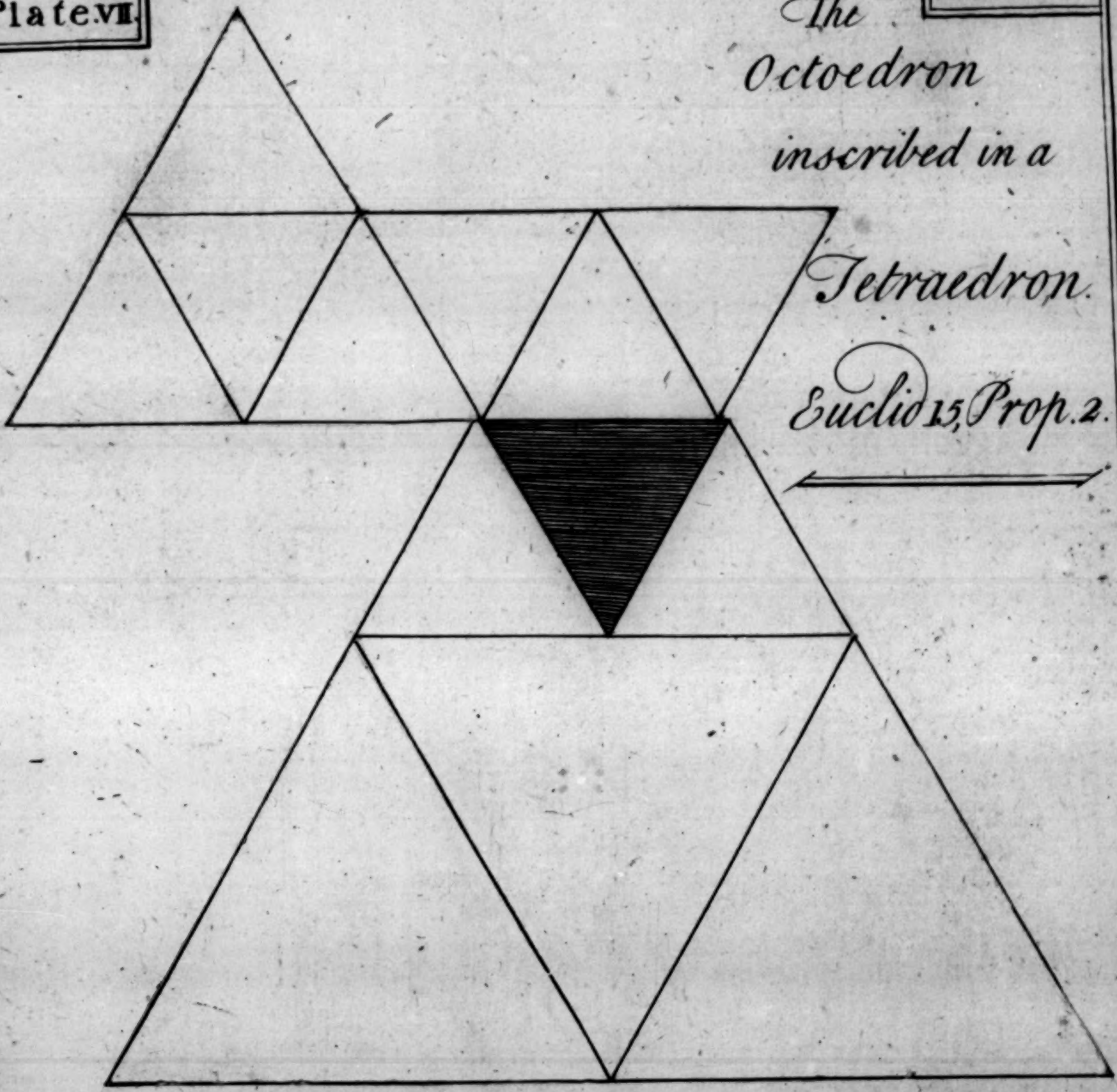


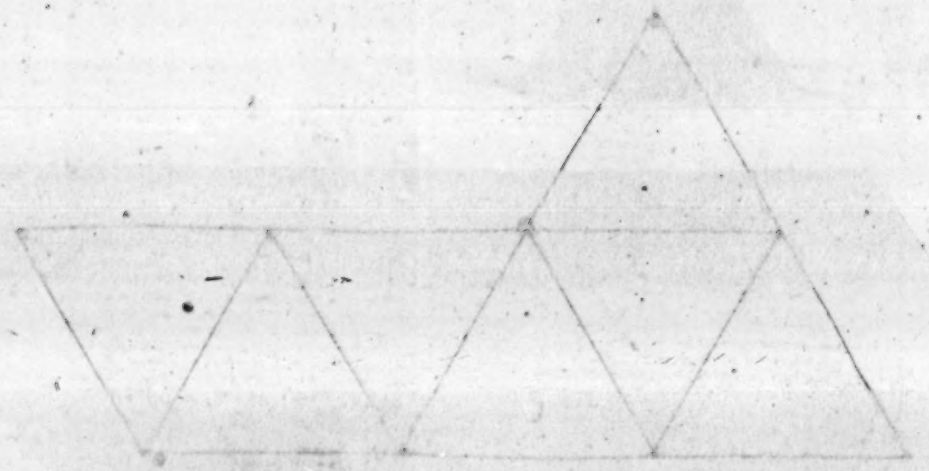
Plate.VII

Book. 2.

The Octoedron inscribed in a Tetraedron.

Euclid 15, Prop. 2.





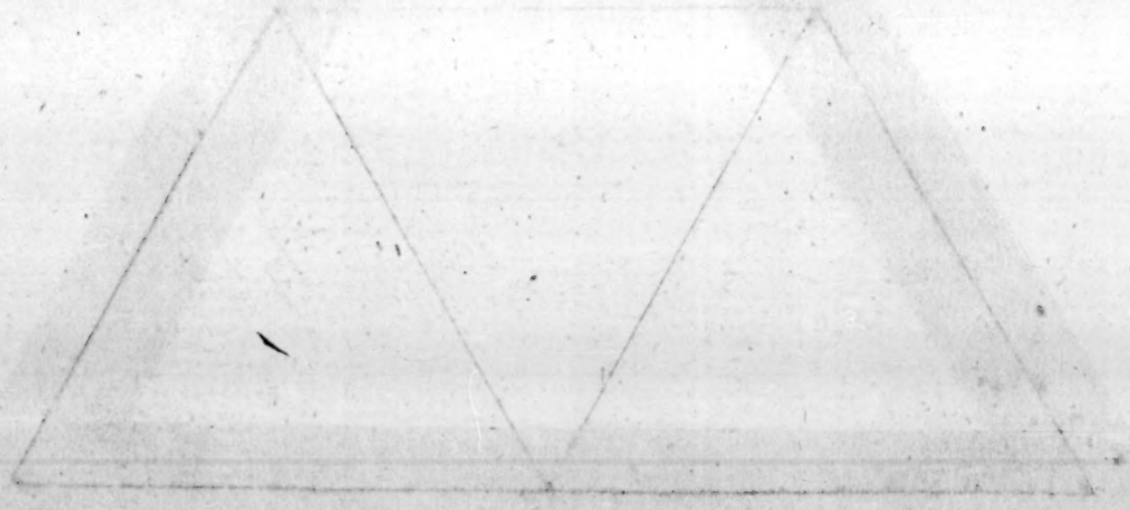


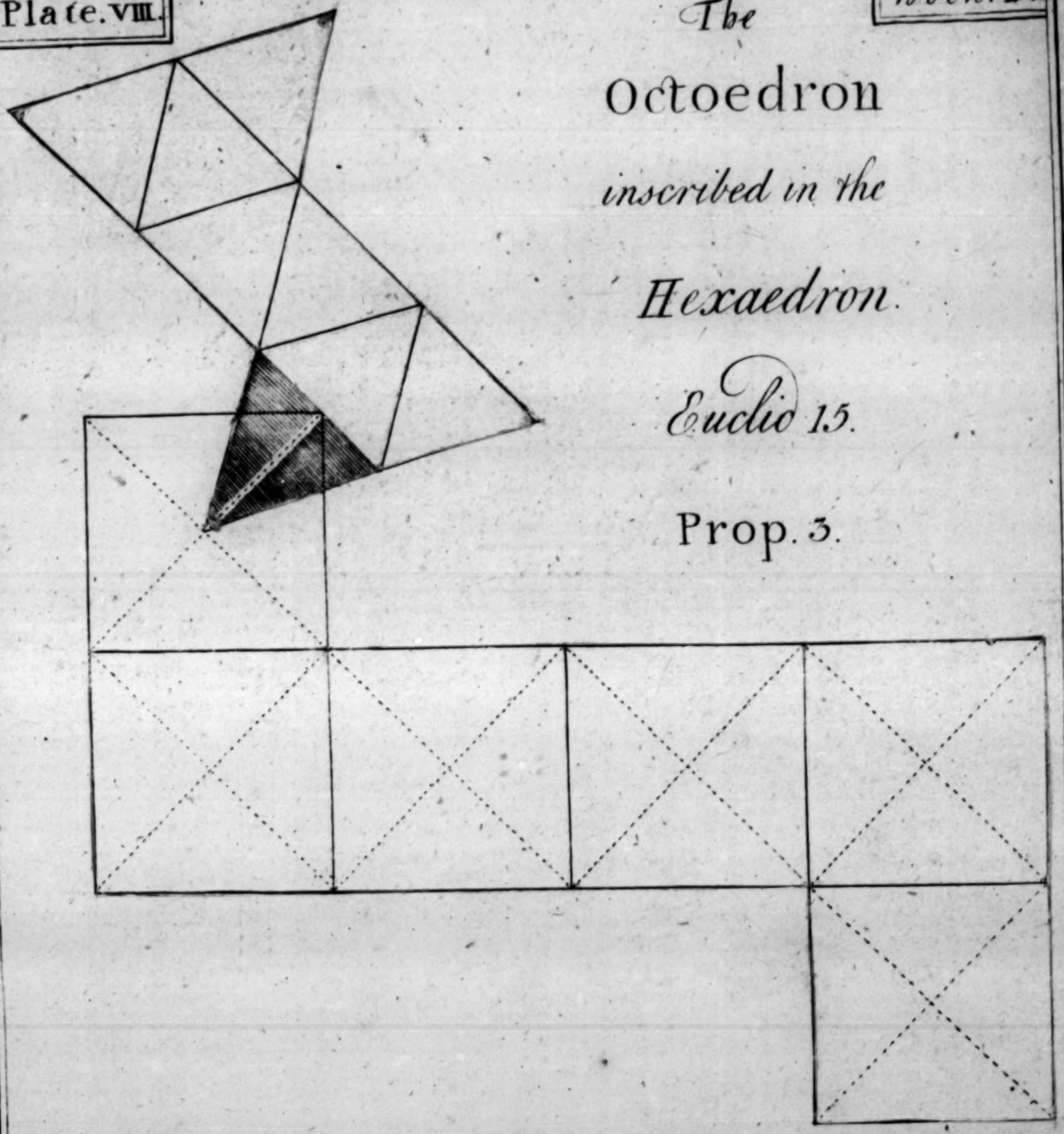
Plate. VIII

Book. 2.

The Octoedron inscribed in the Hexaedron.

Euclid 15.

Prop. 3.



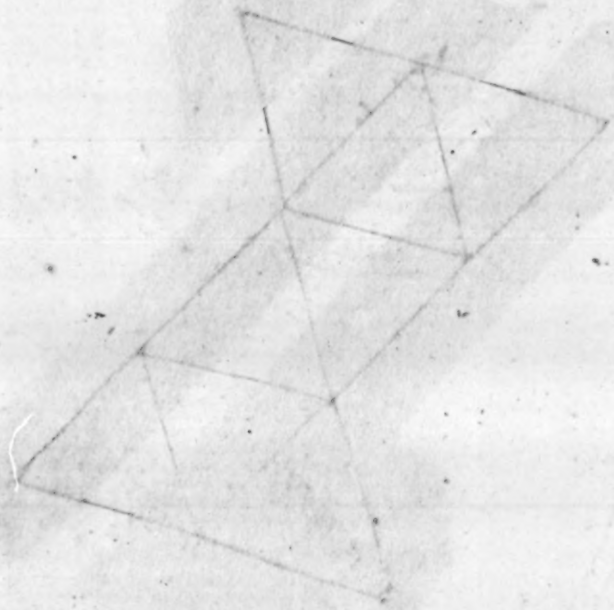


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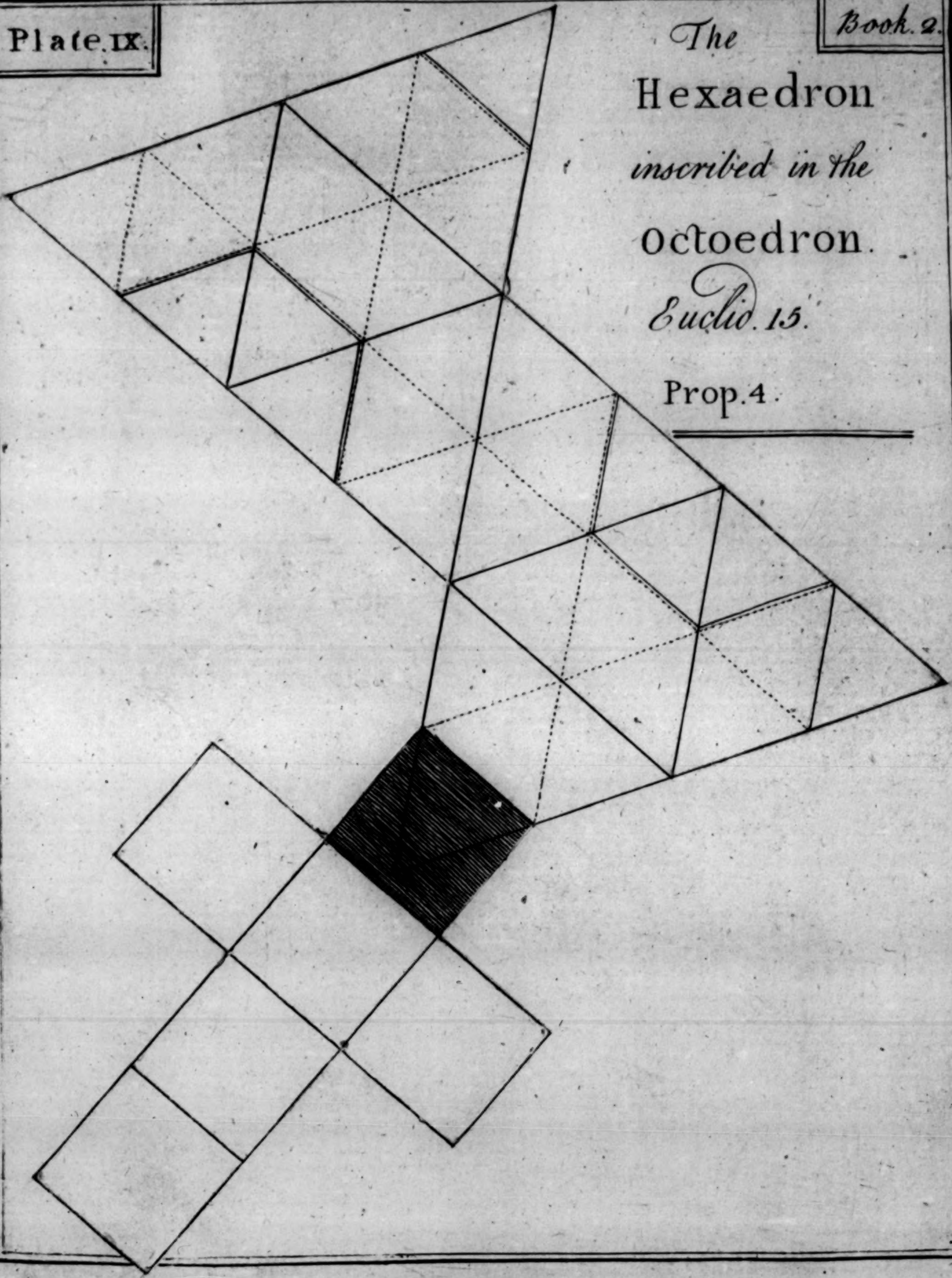
Plate.IX.

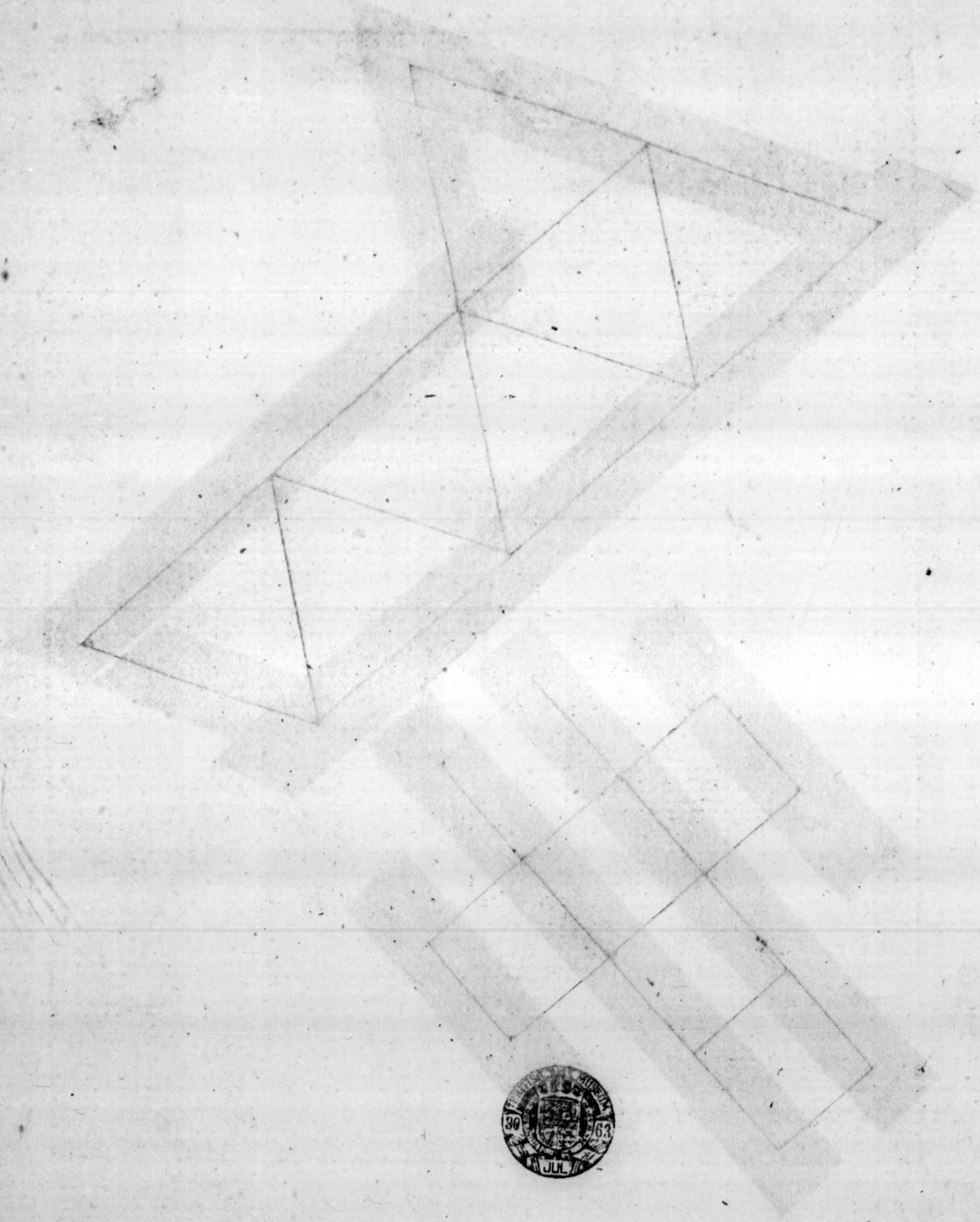
Book. 2.

The Hexaedron inscribed in the Octoedron.

Euclid. 15.

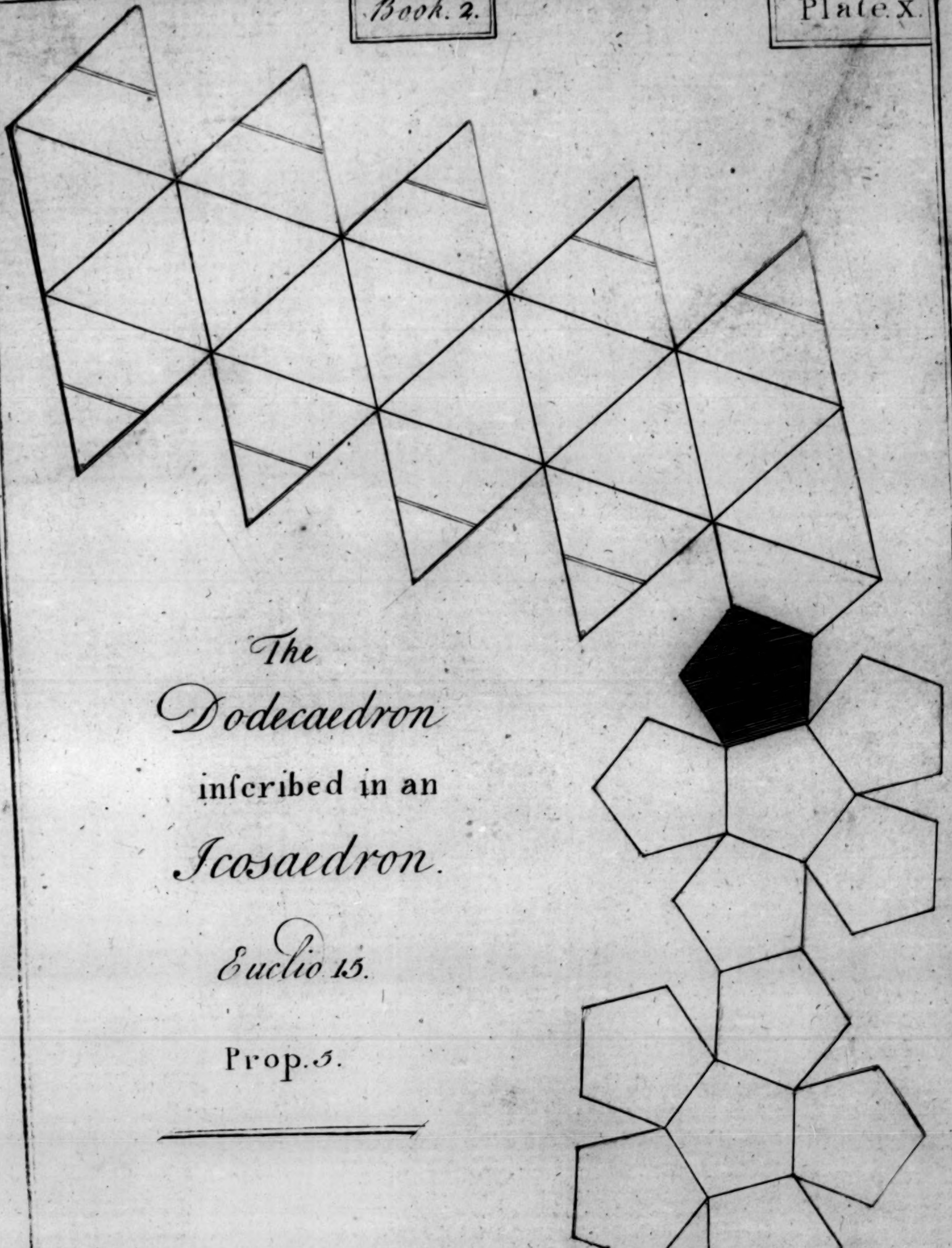
Prop. 4.





Book. 2.

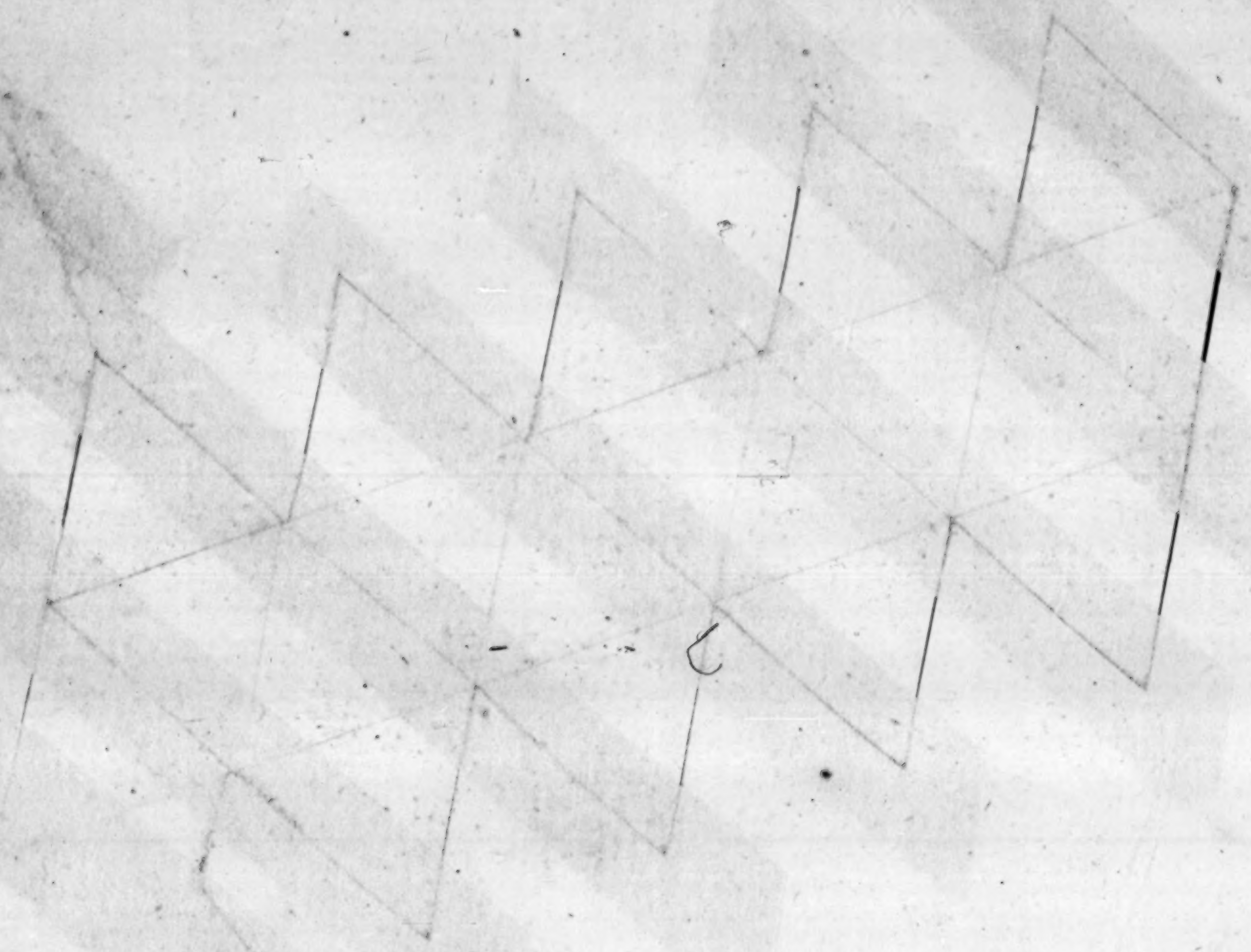
Plate X.

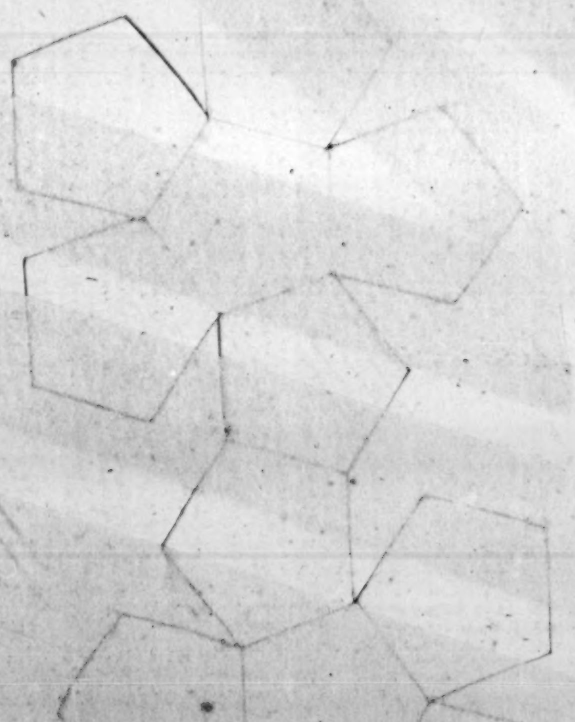


The Dodecaedron inſcribed in an Icoſaedron.

Euclid 15.

Prop. 5.





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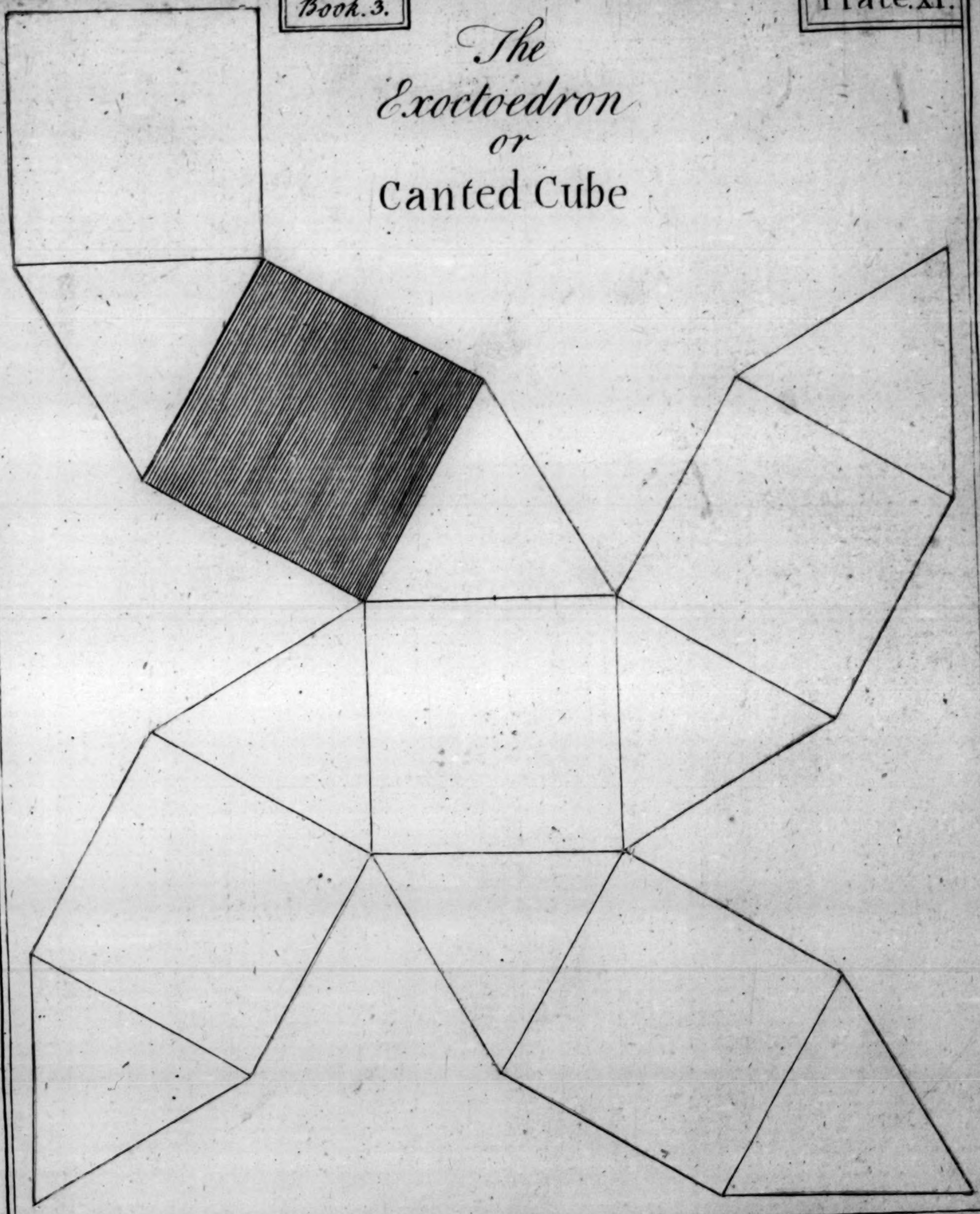
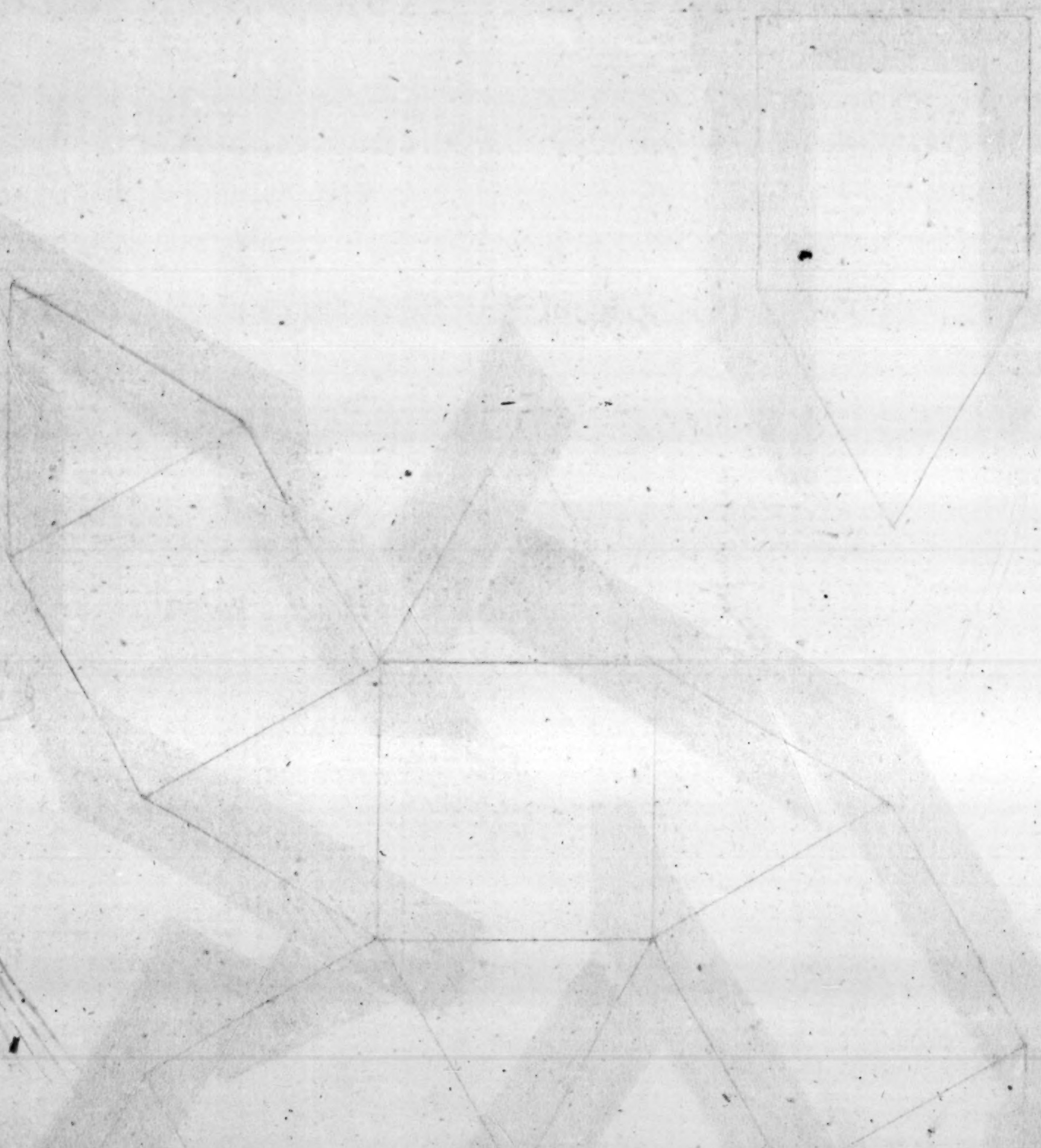
Book. 3.

Plate. XI.

The Exoctoedron or Canted Cube





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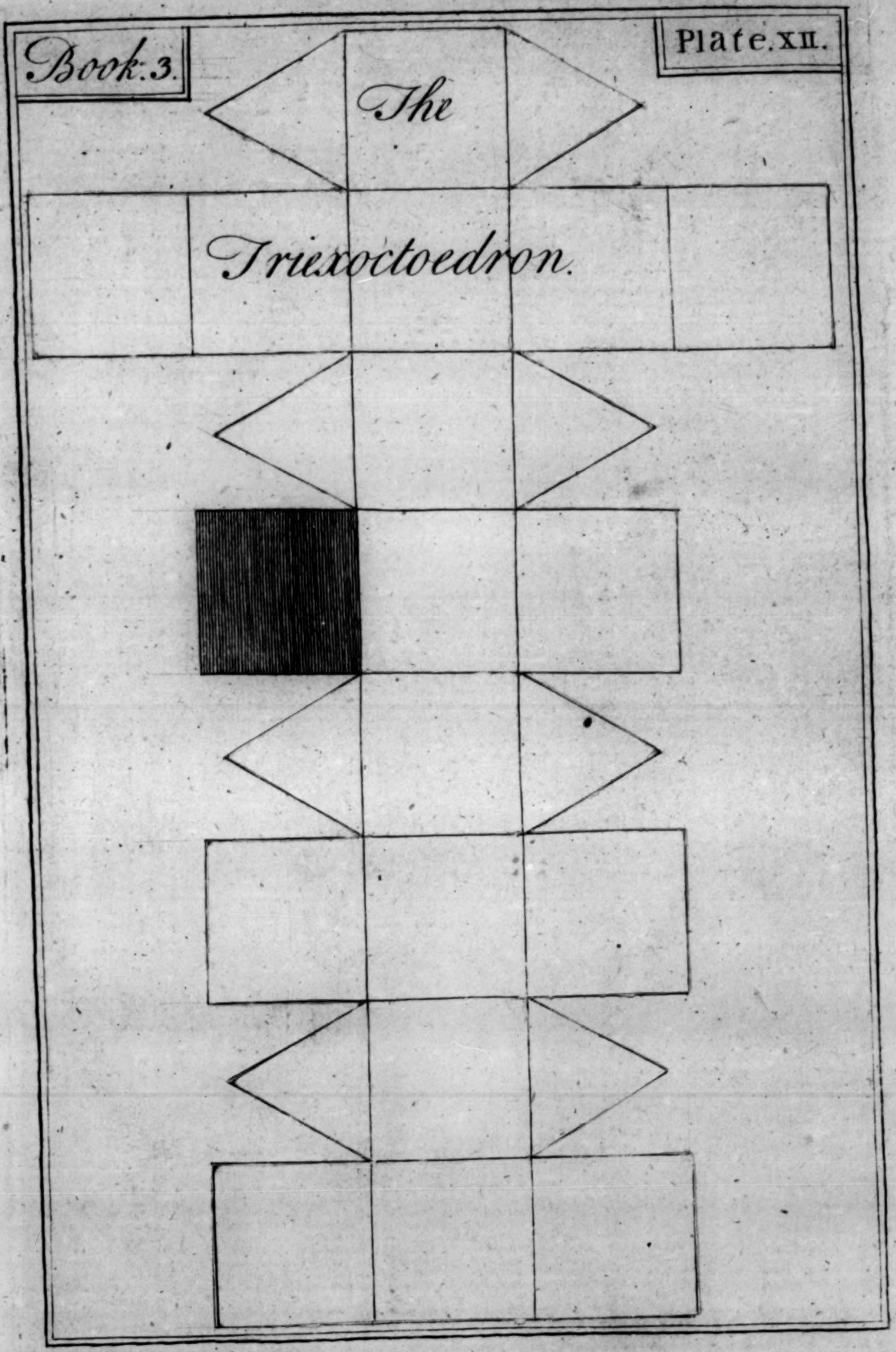
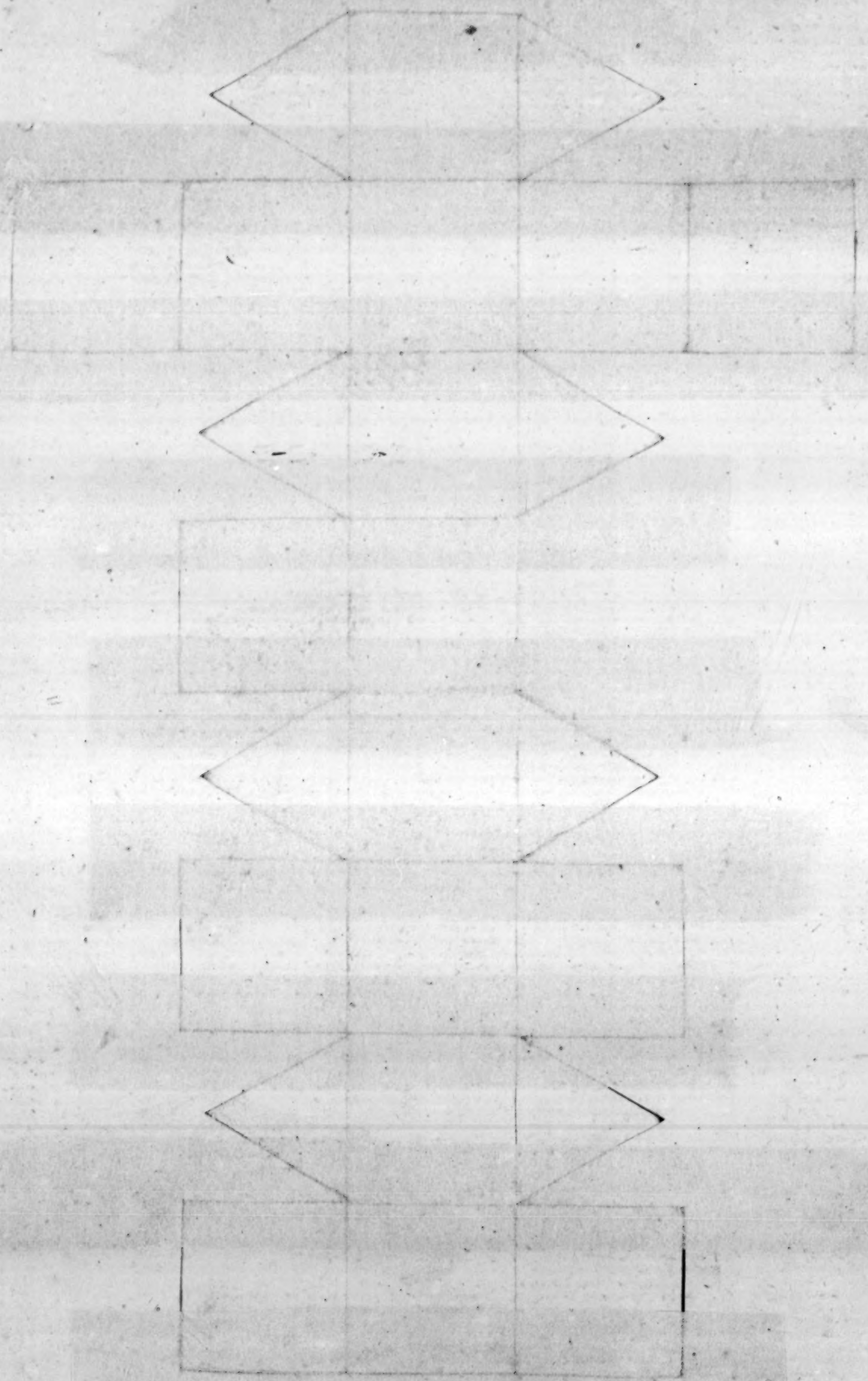
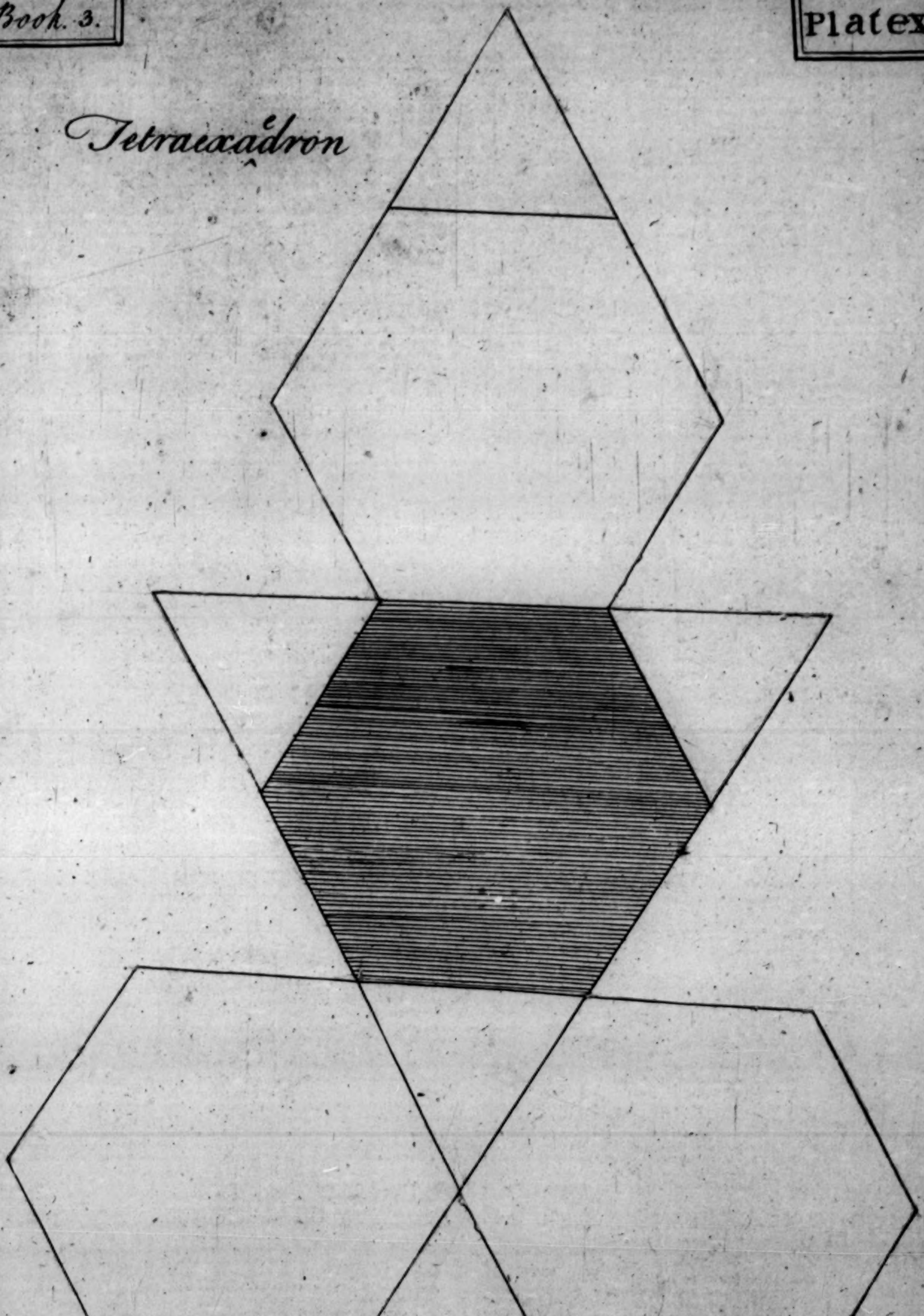
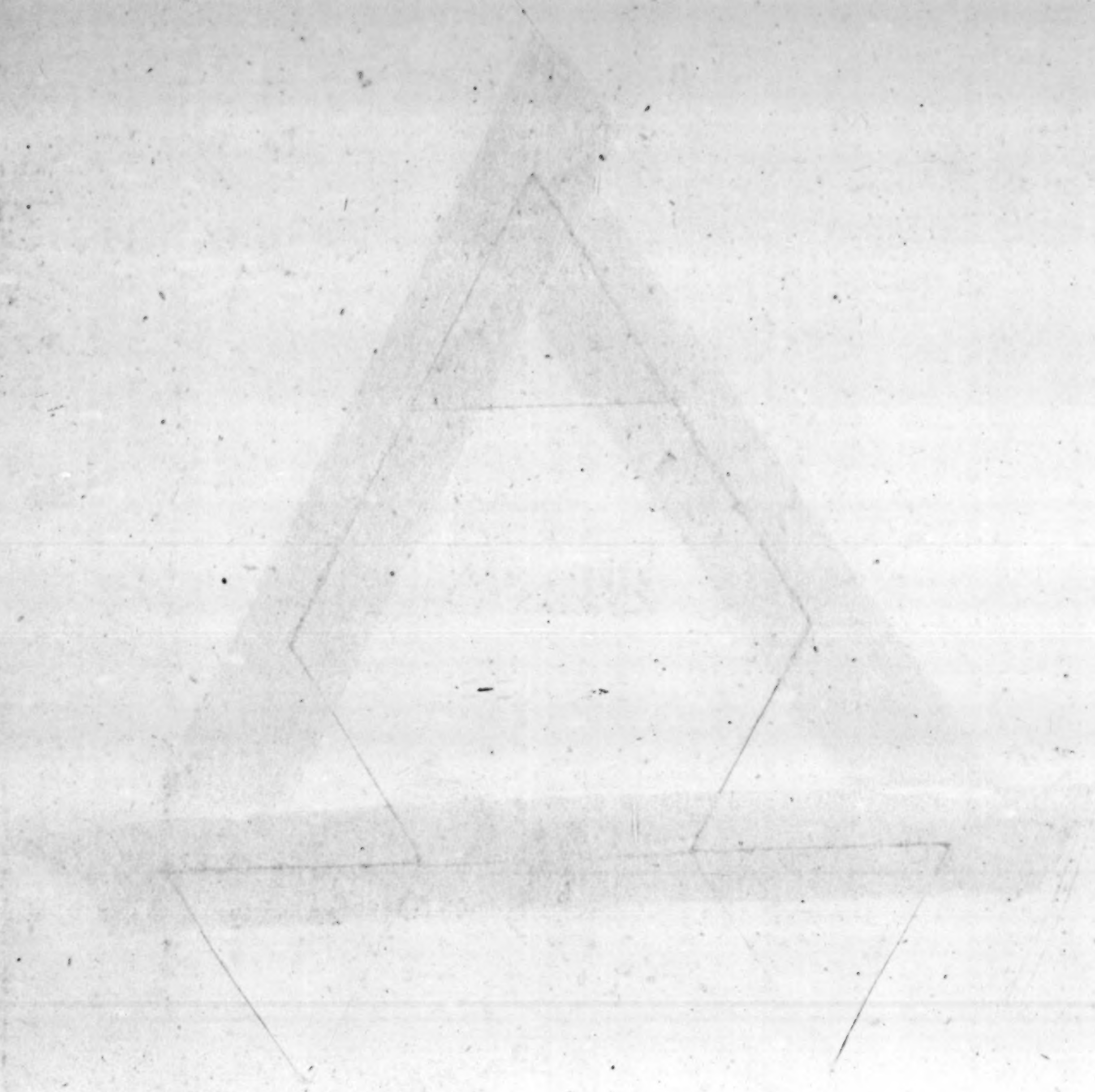

Book:3.
Plate.XII.
The
Triexoctoedron.



Tetraexaedron





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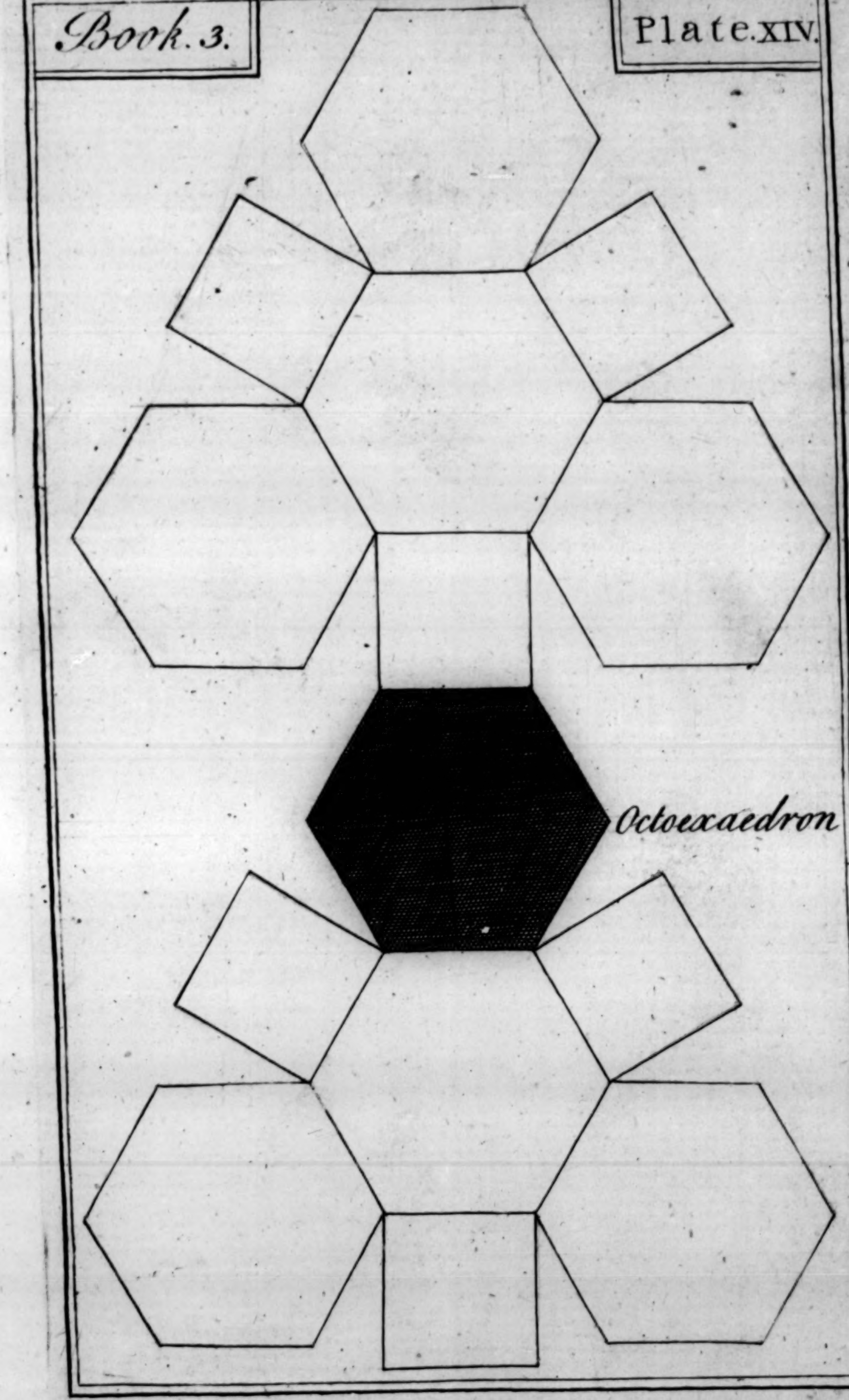
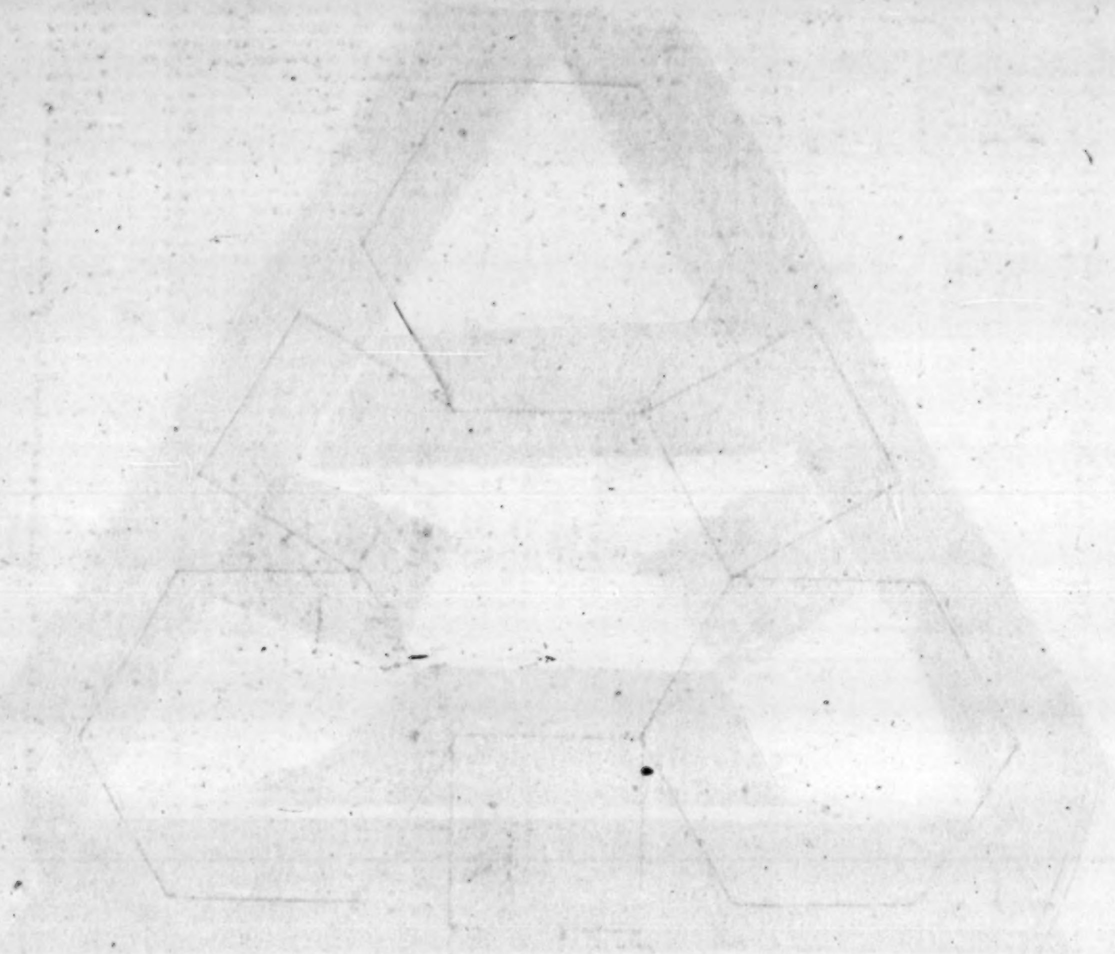
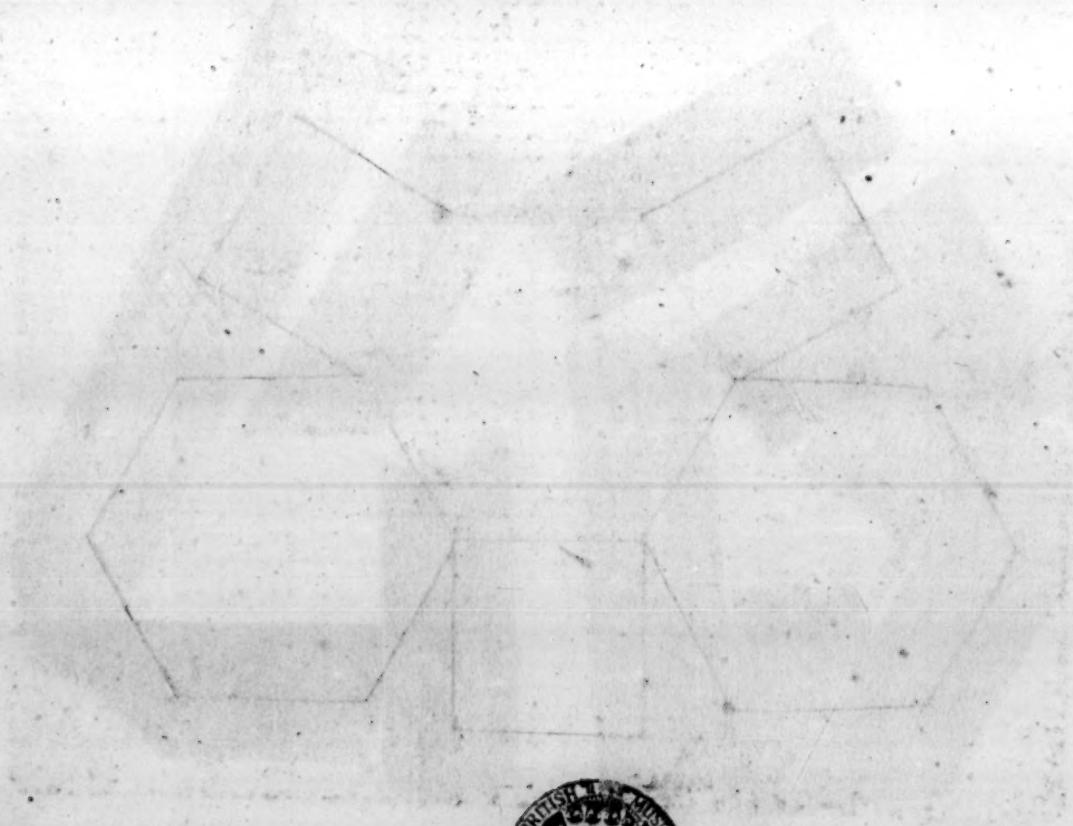
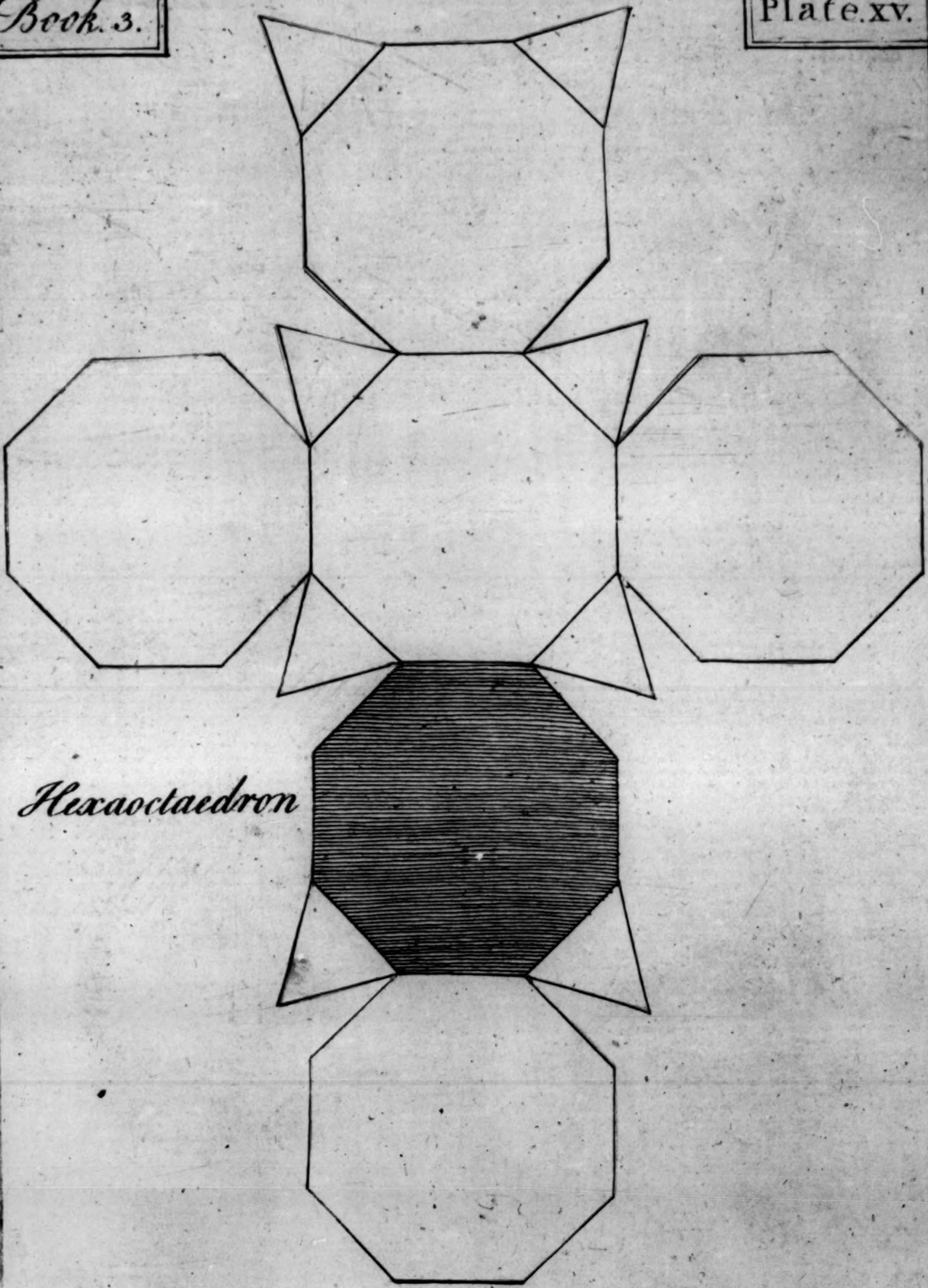
Book. 3.

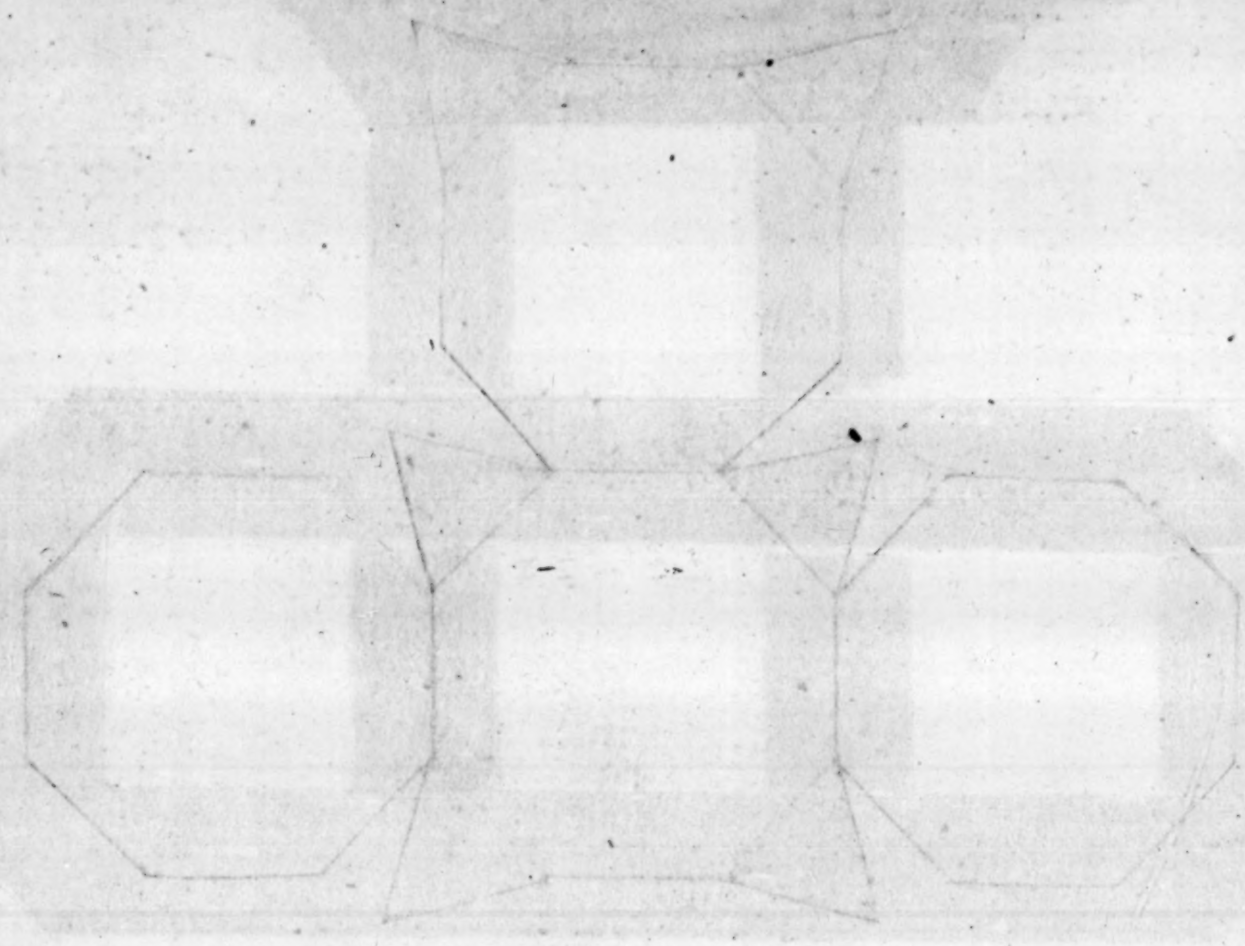
Plate. XIV.

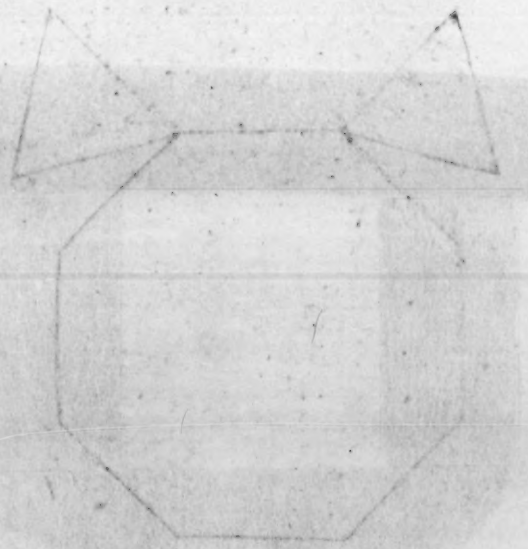







Hexaoctaedron

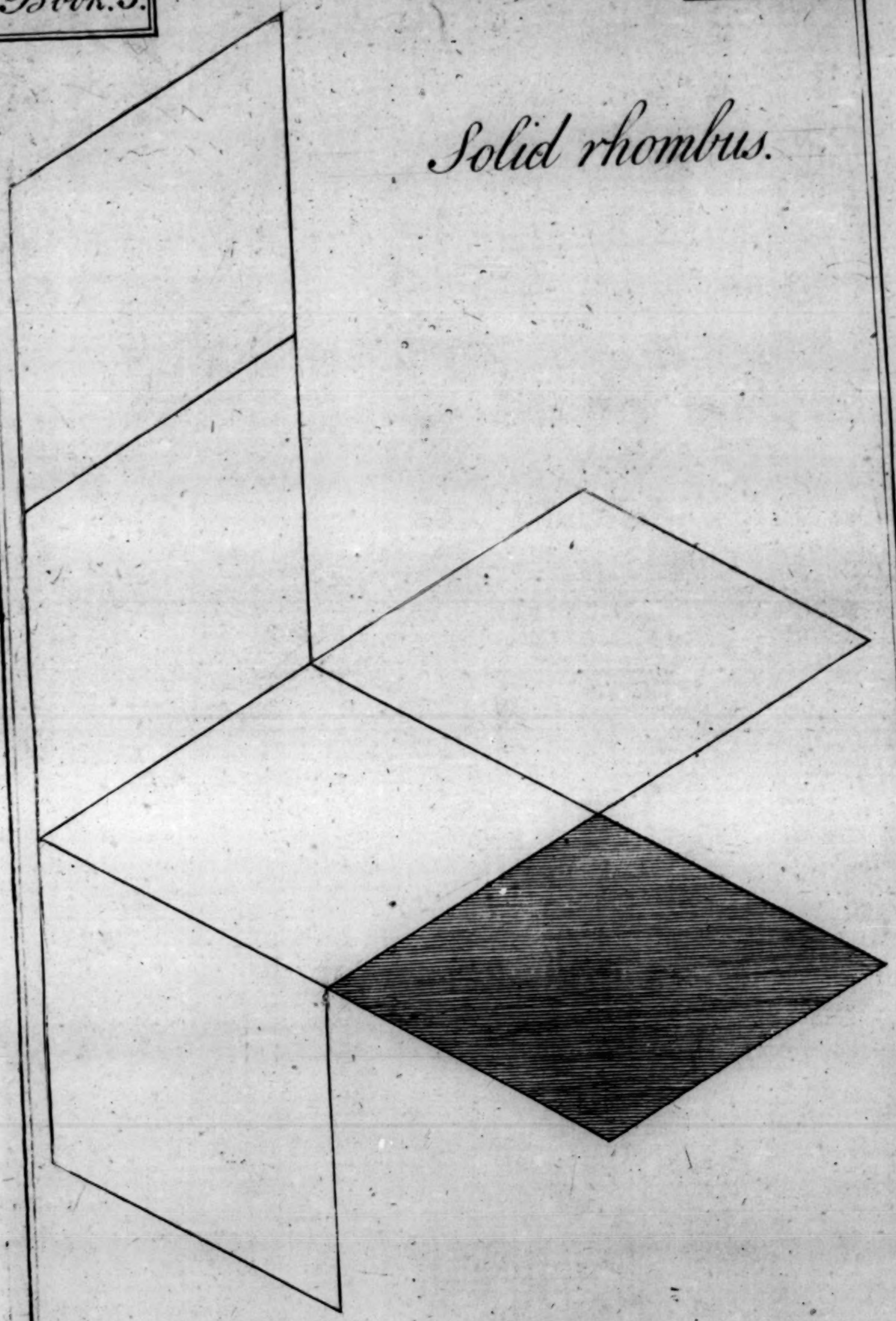


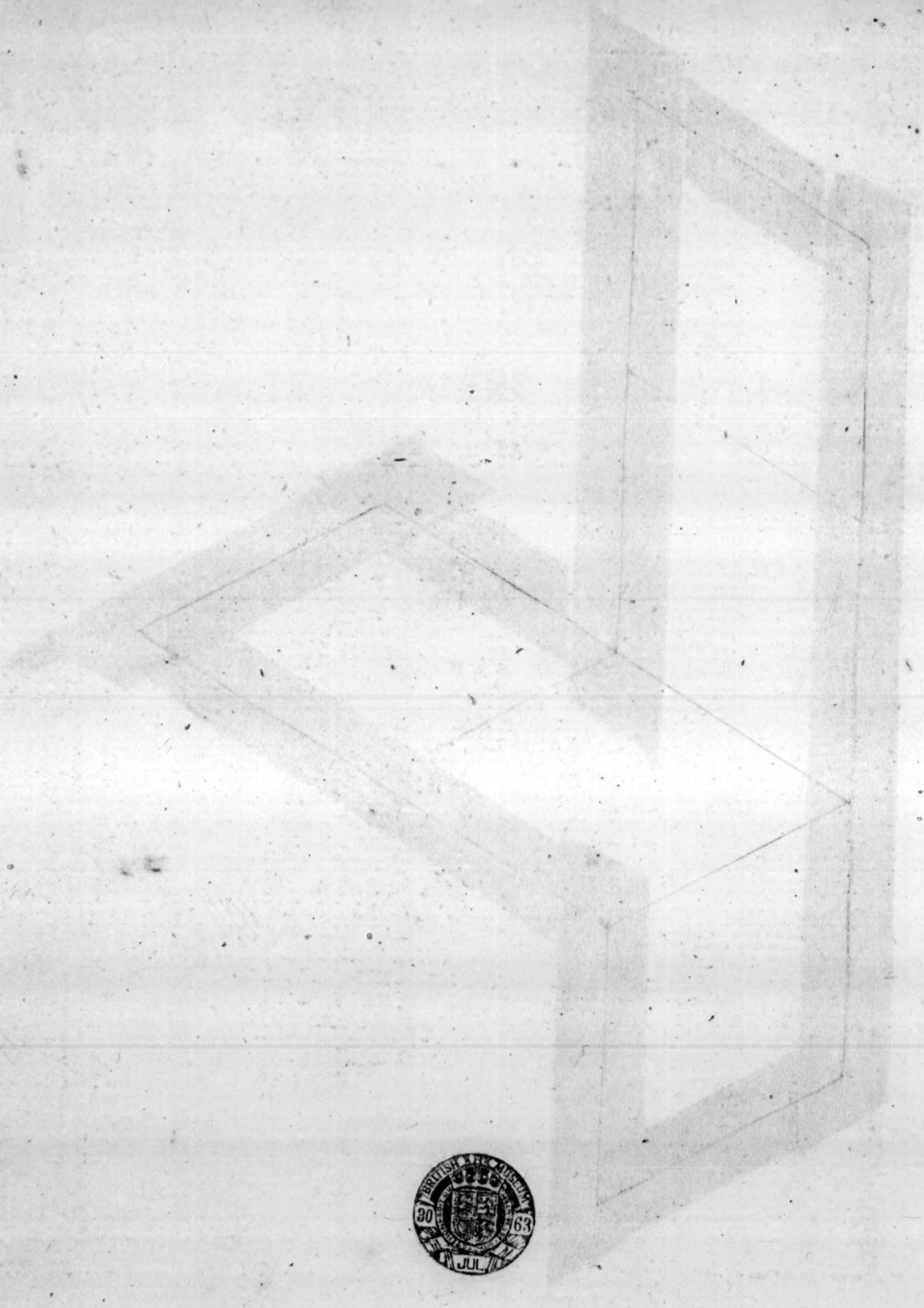


Book. 3.

Plate XVI

Solid rhombus.

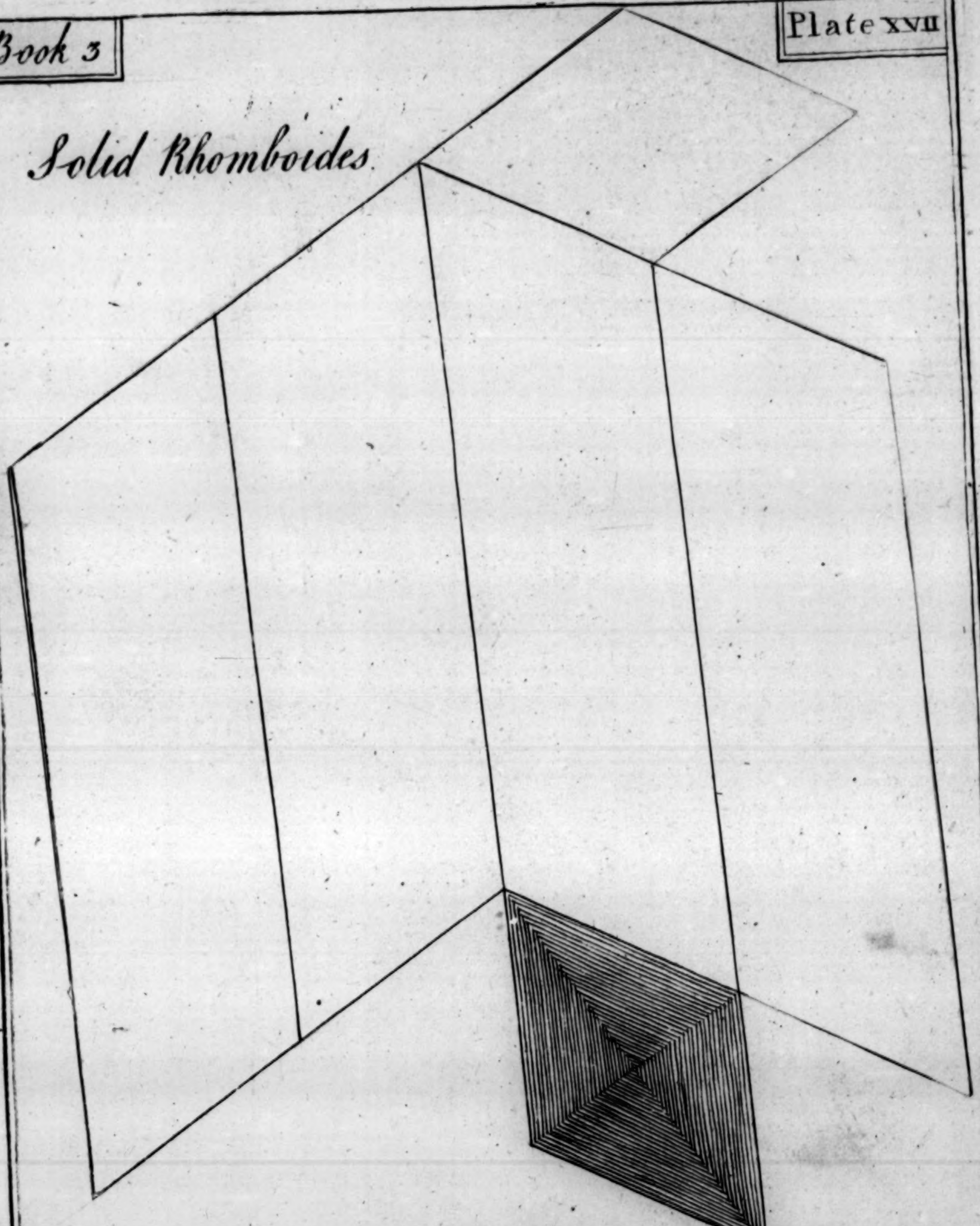


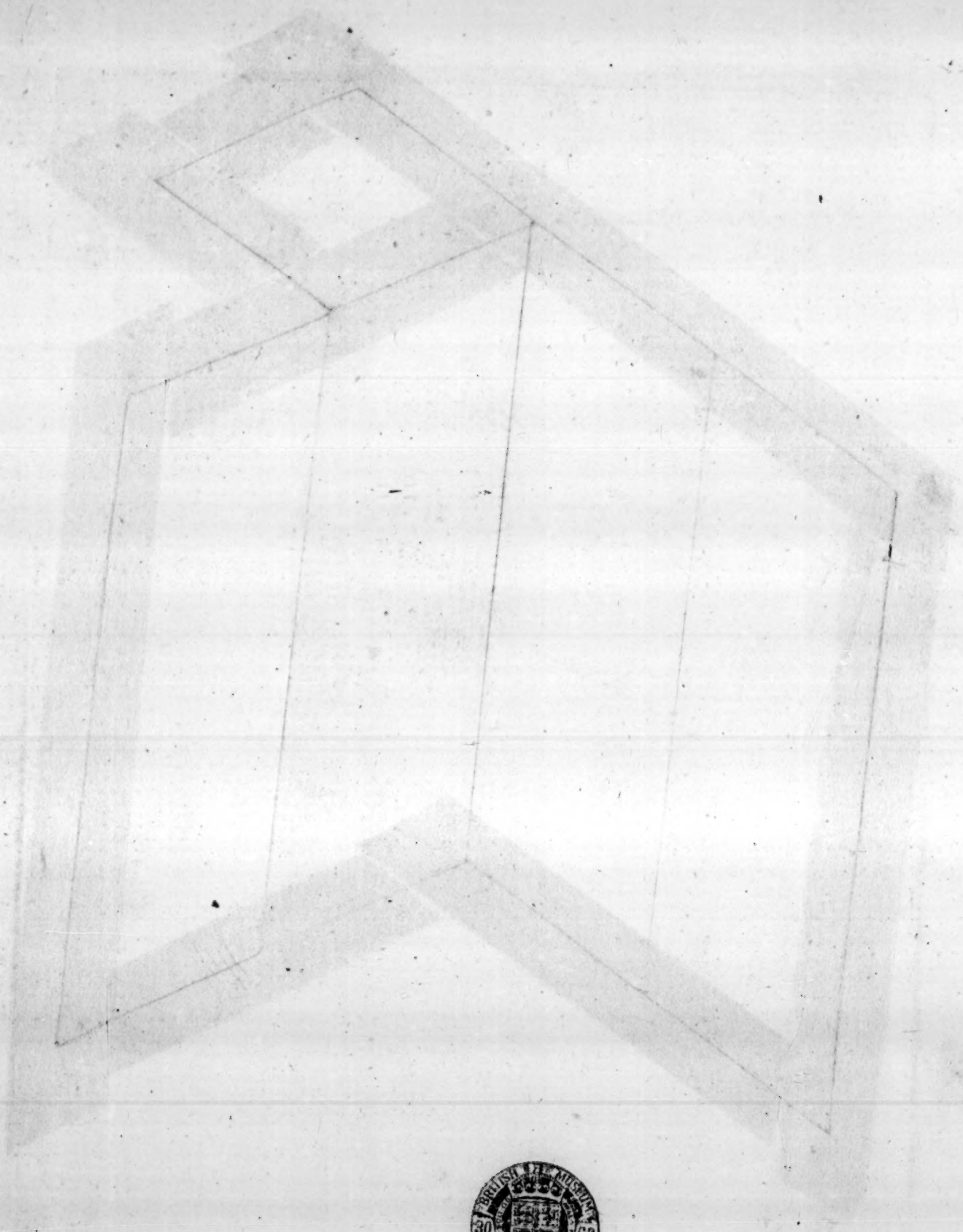


Book 3

Plate XVII

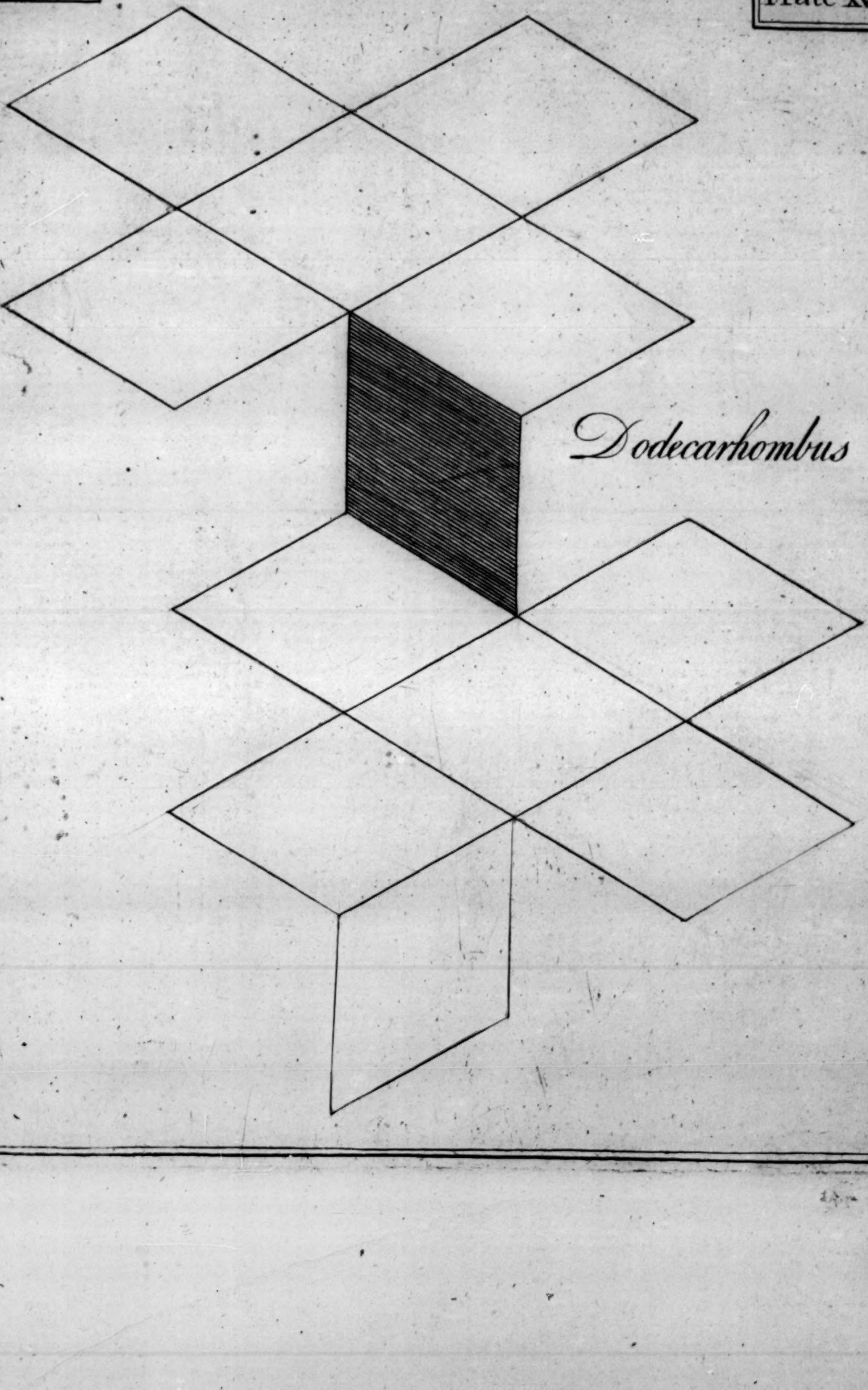
Solid Rhomboides.





Book.3.

Plate XVIII



Dodecarhombus

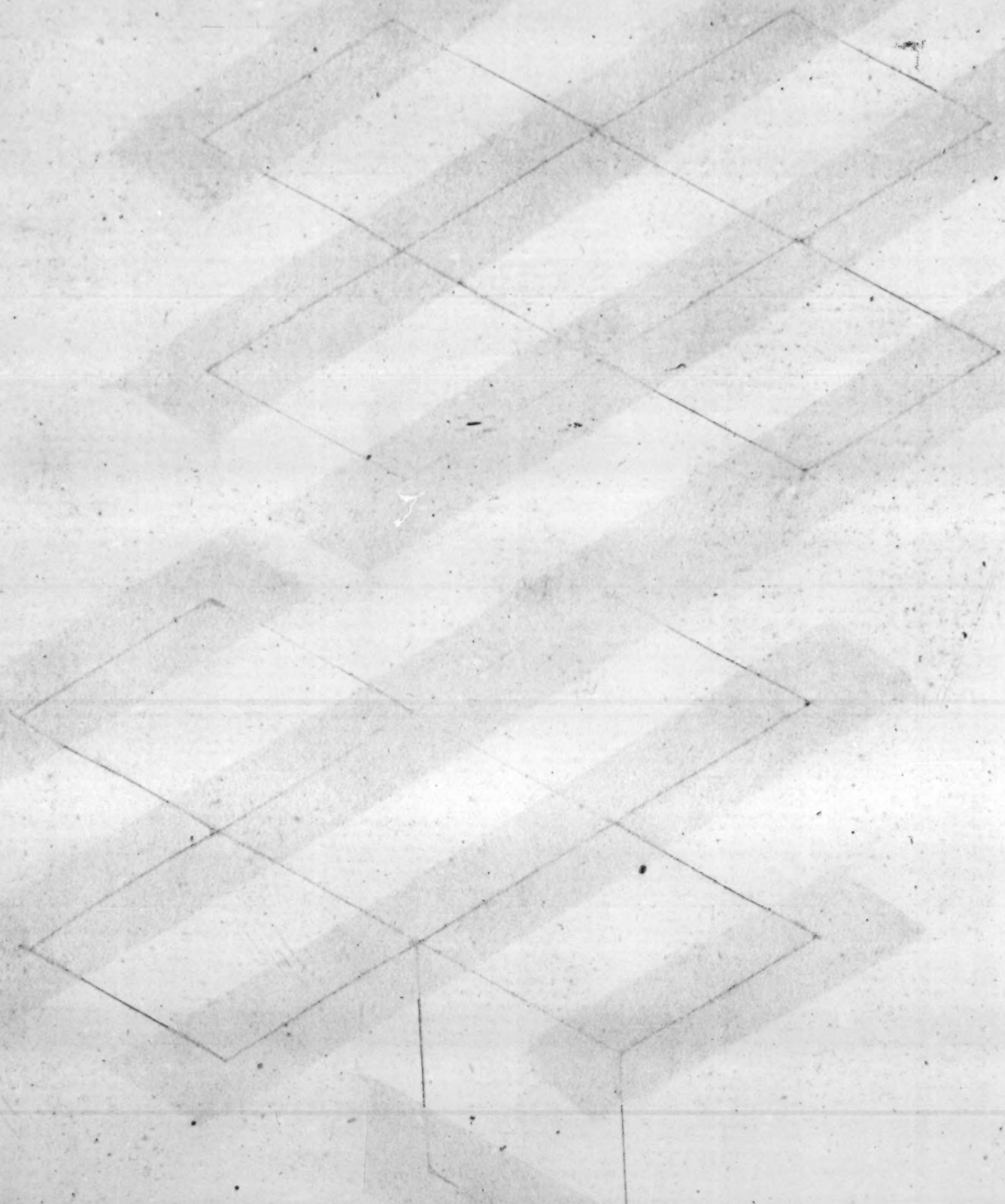
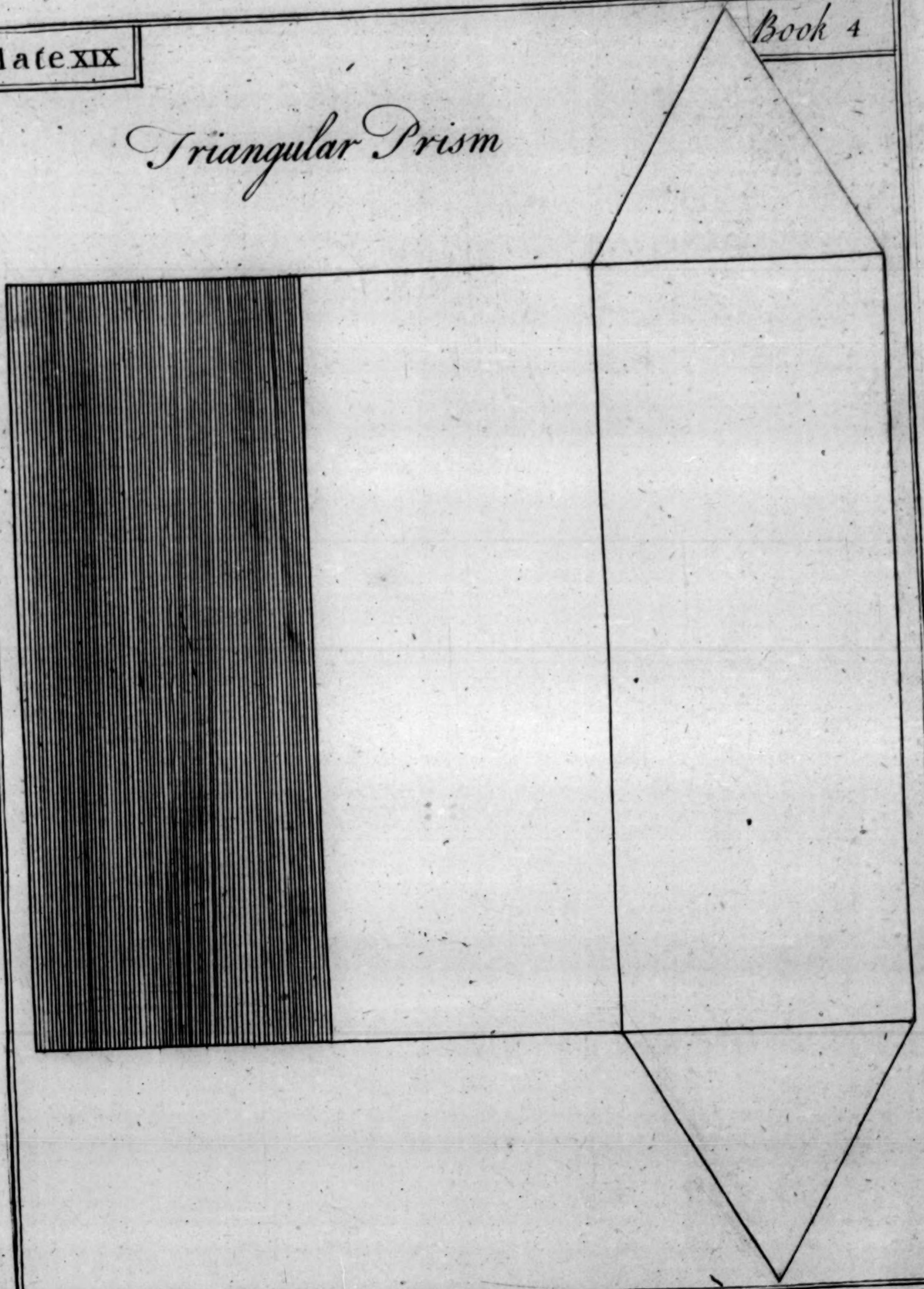
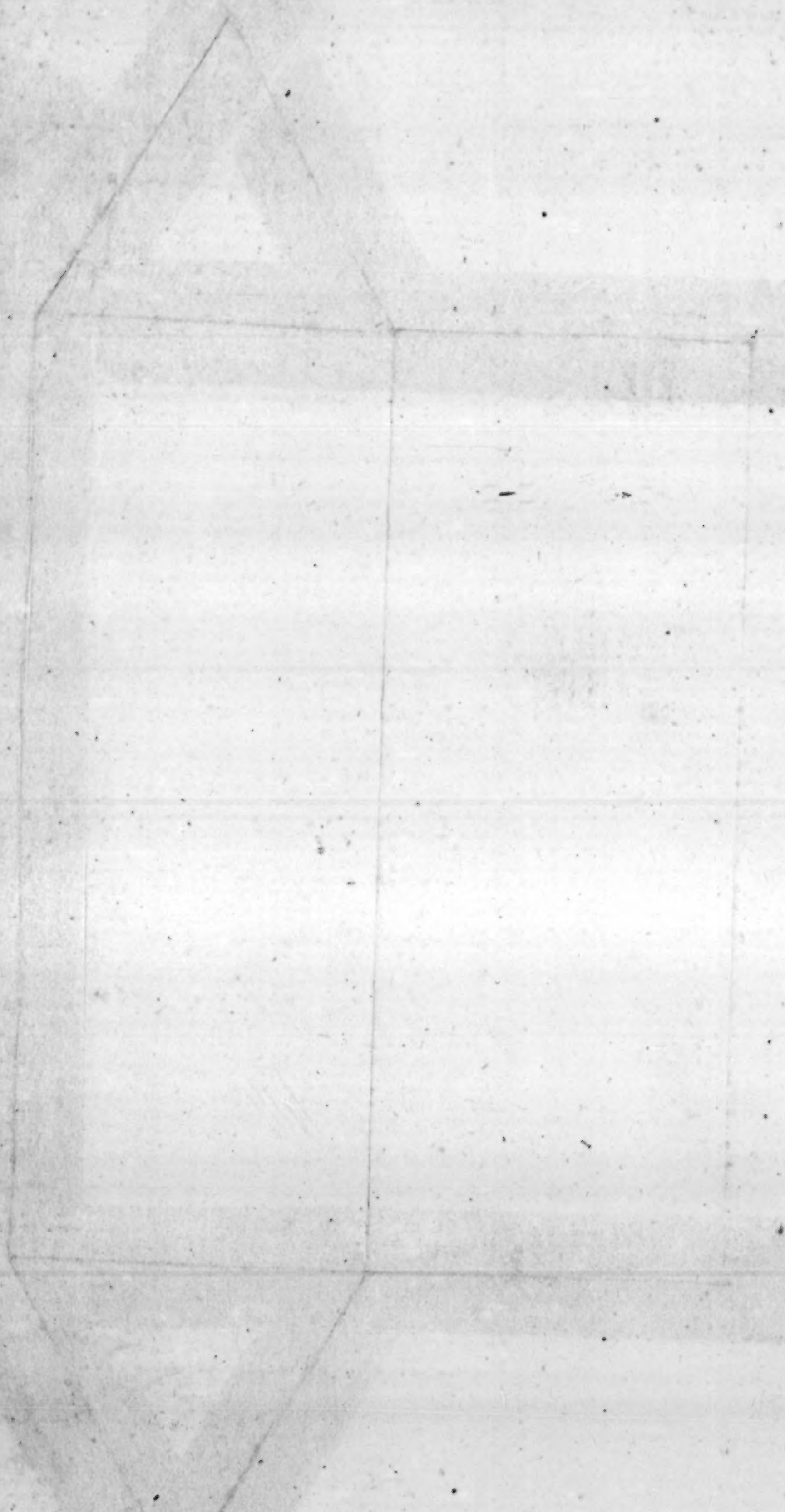


Plate XIX

Book 4

Triangular Prism

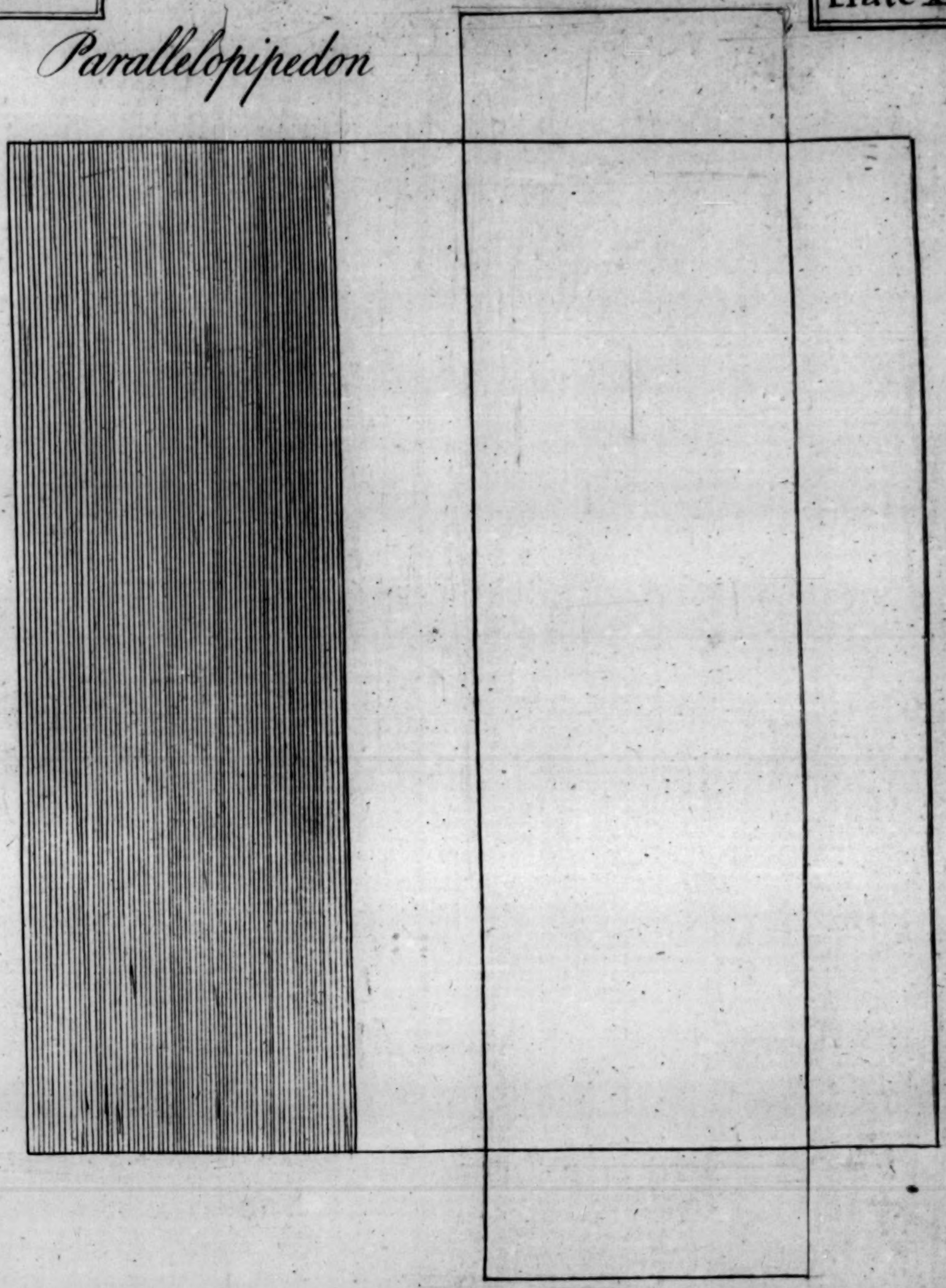




Book. 4.

Plate XX

Parallelopipedon



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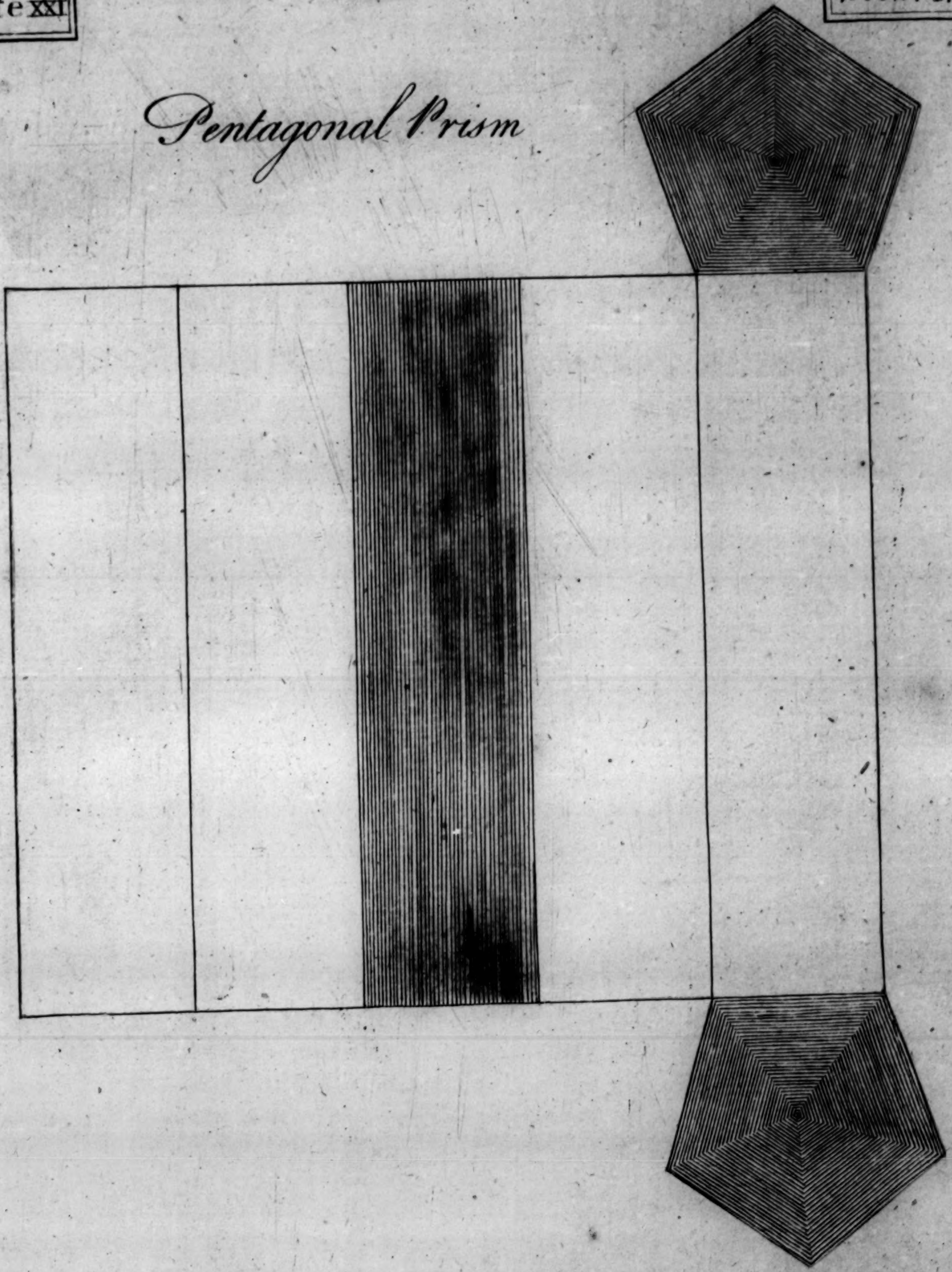
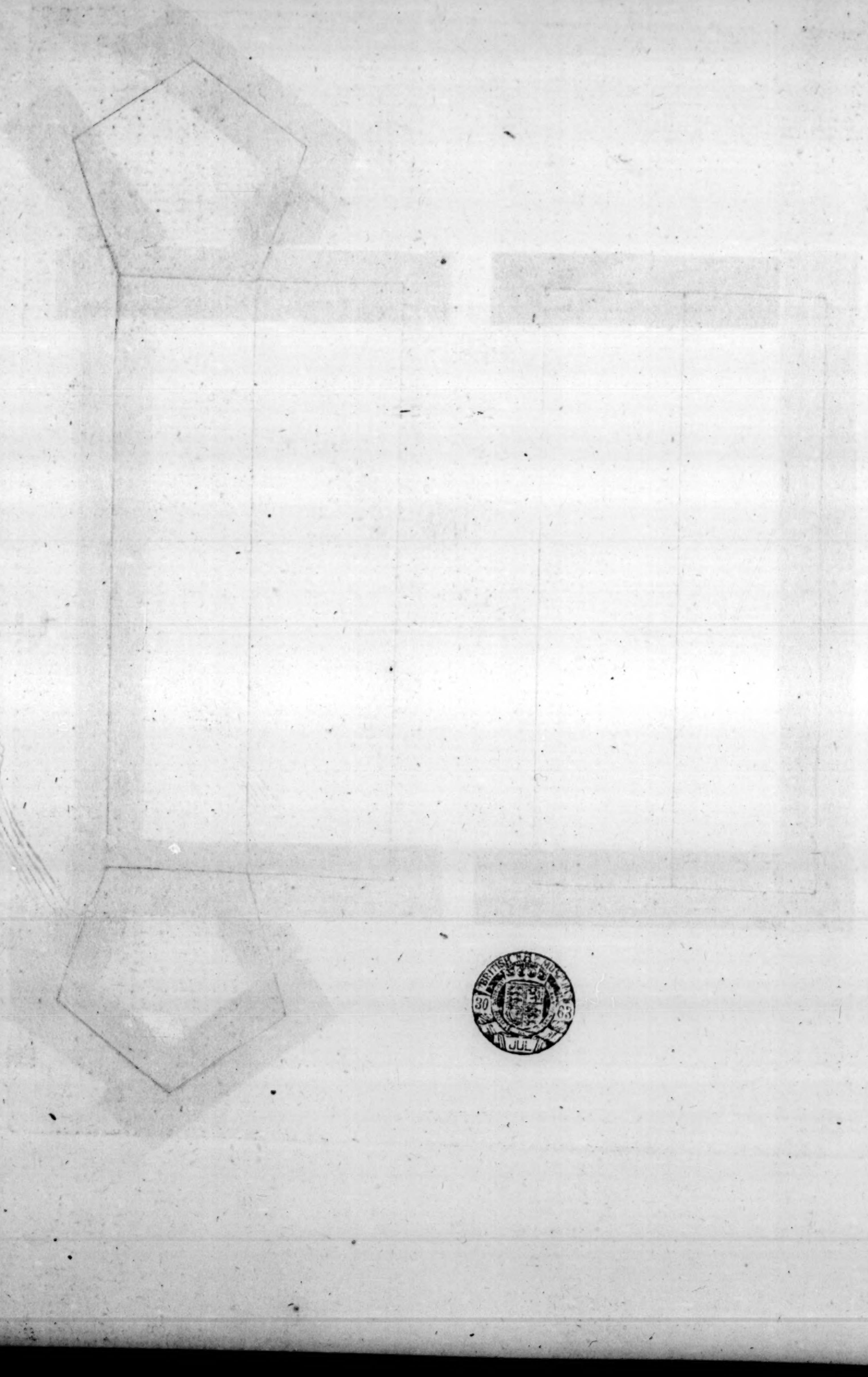
Pentagonal Prism





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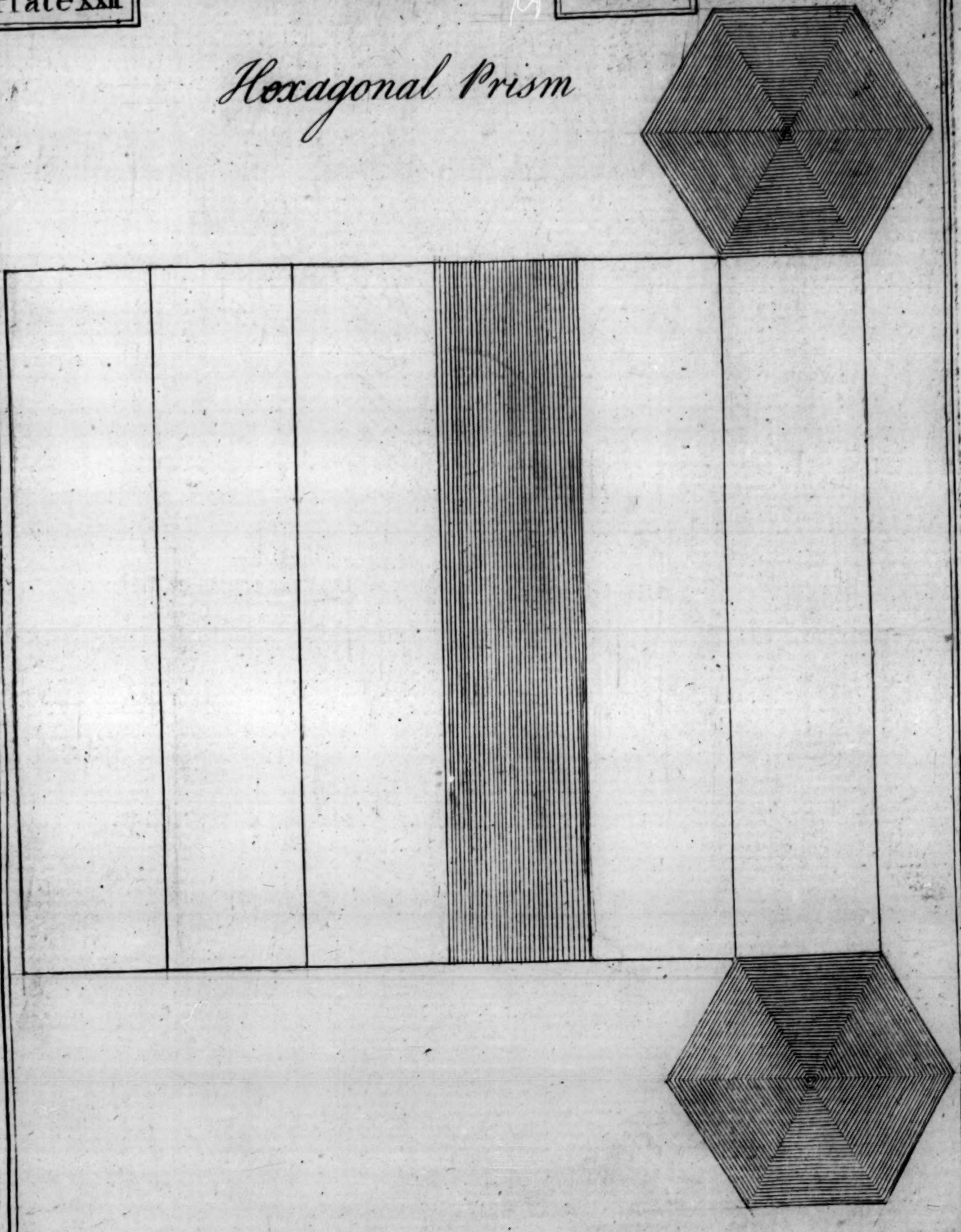
Hexagonal Prism



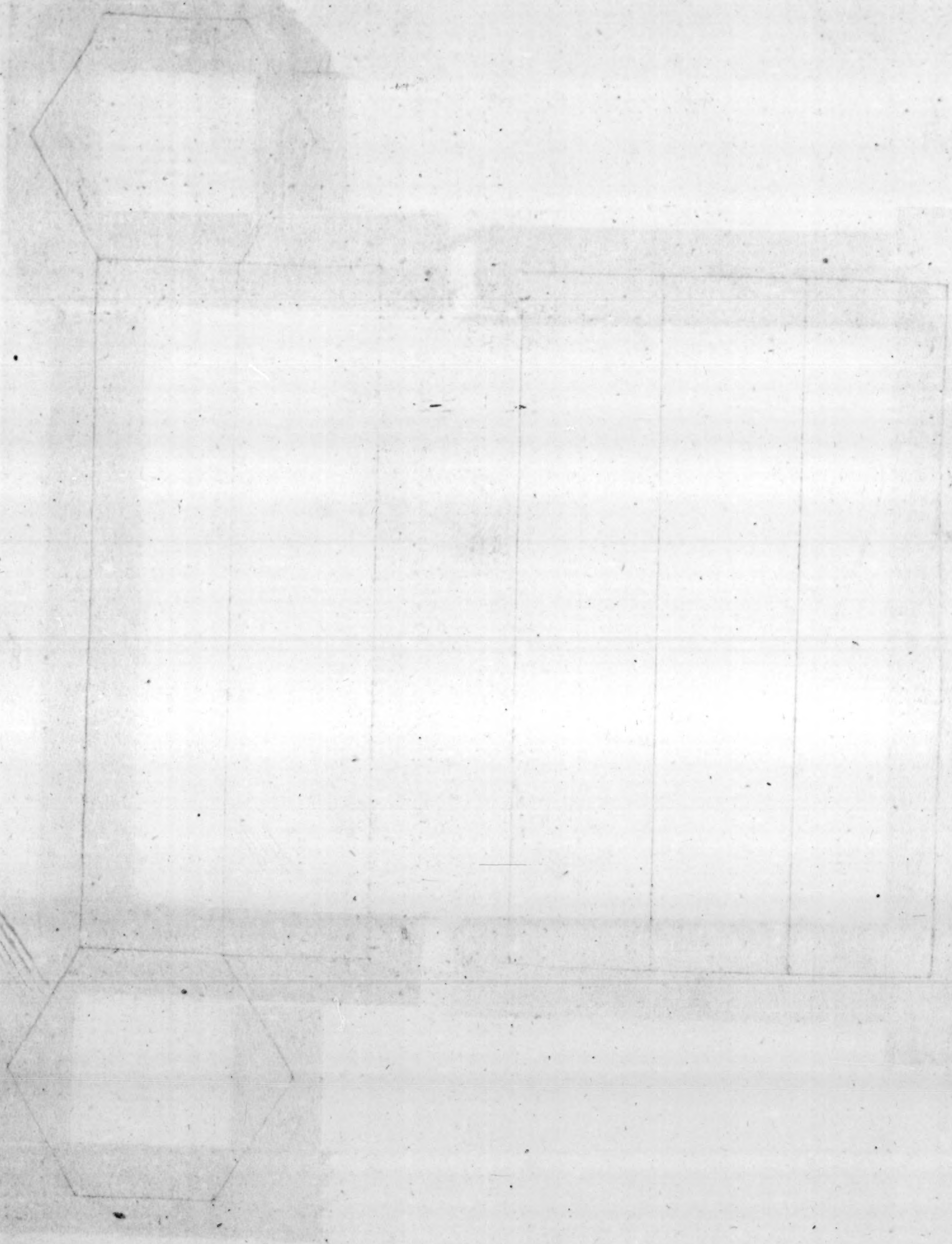
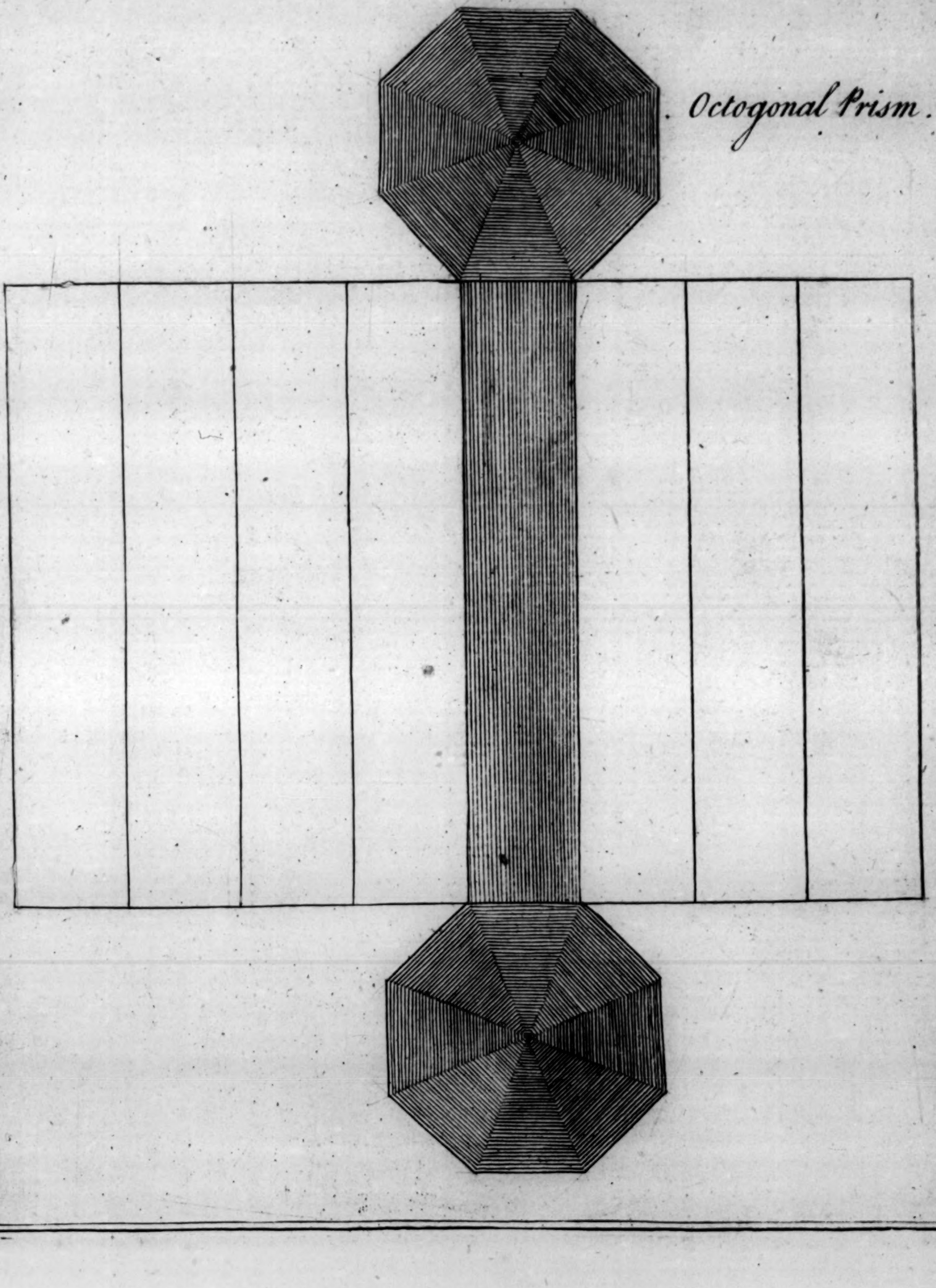


Plate XXIII

Book. 4.



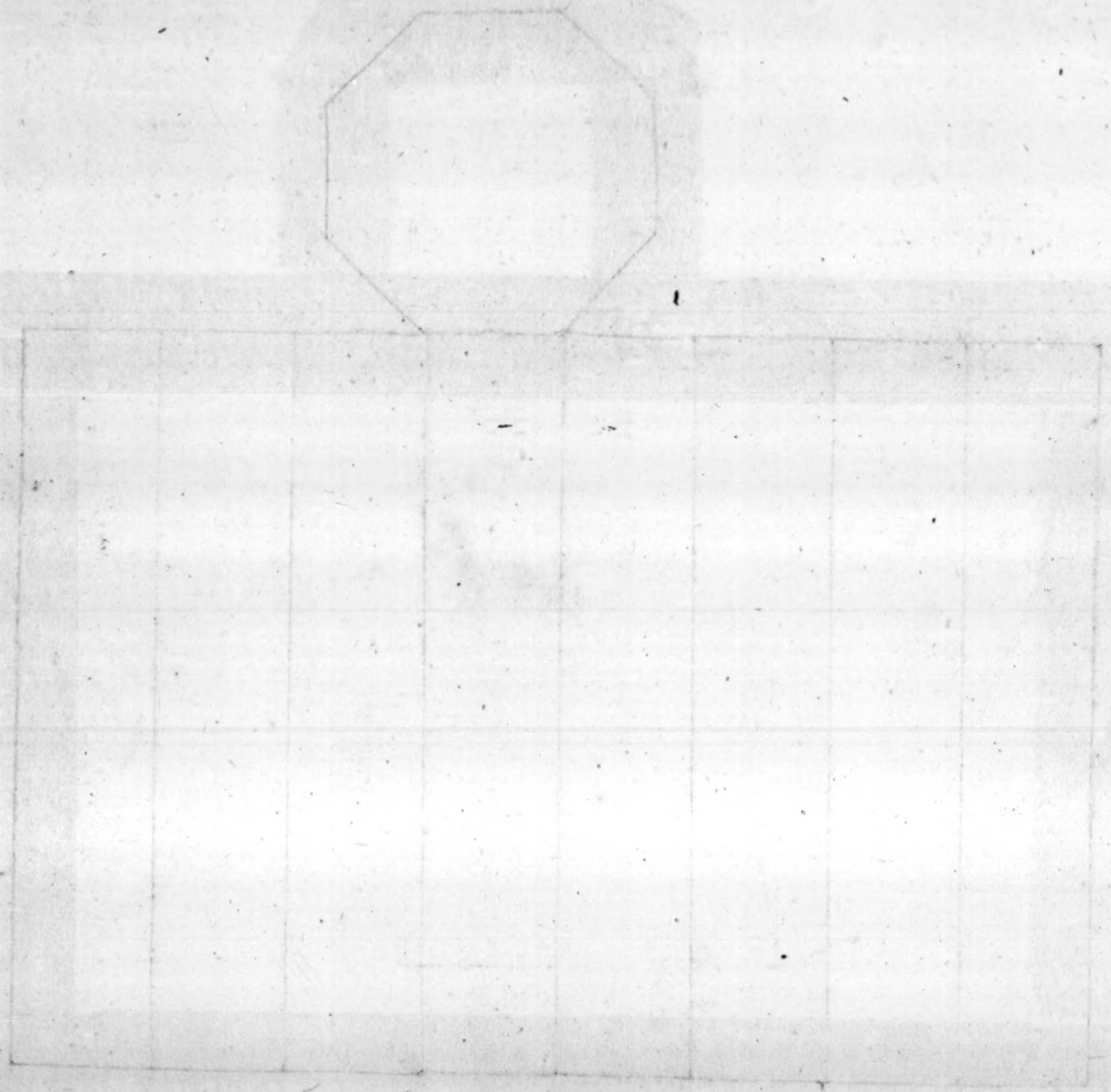


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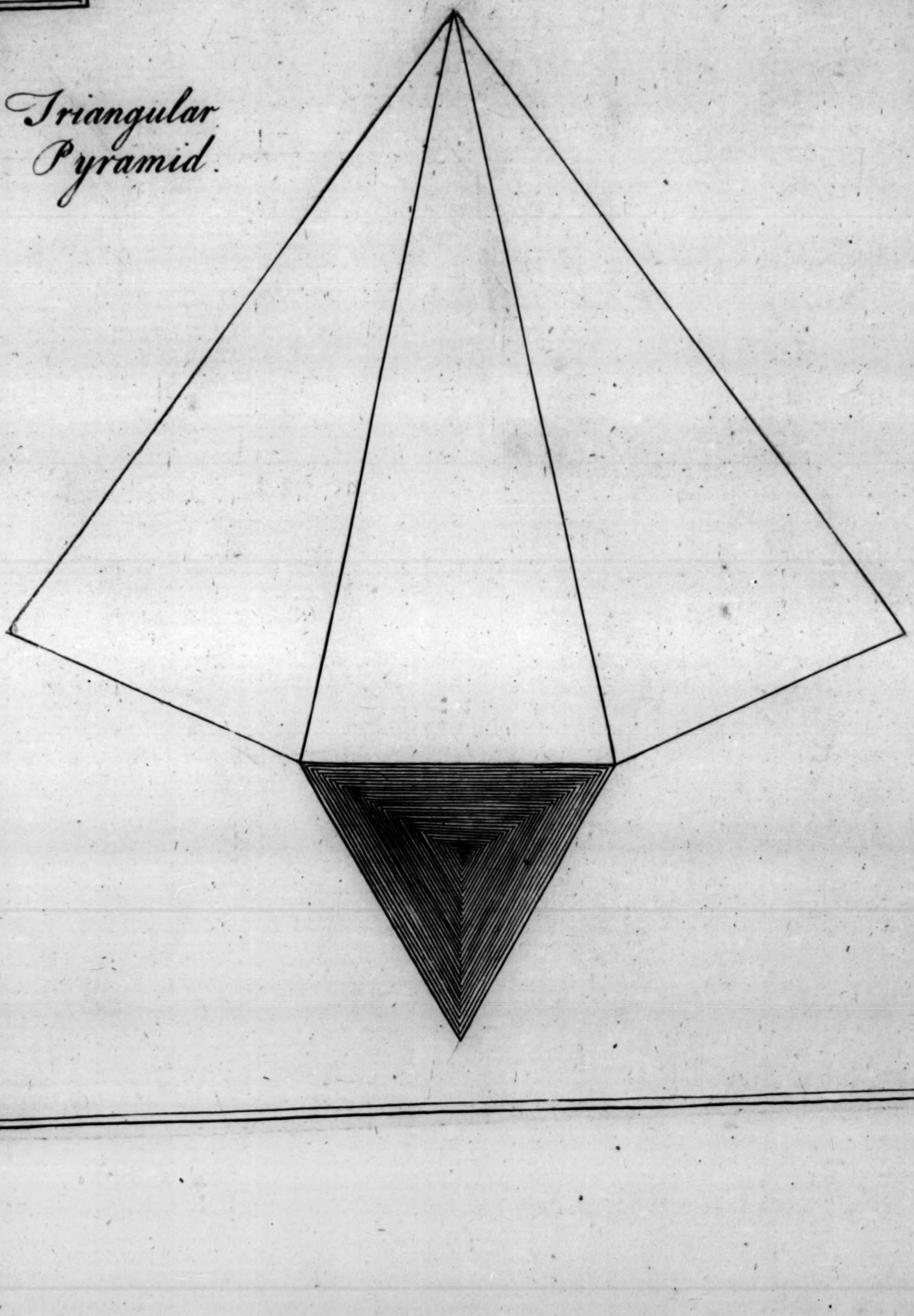
Plate XXIV

Book. 5.

Triangular Pyramid.



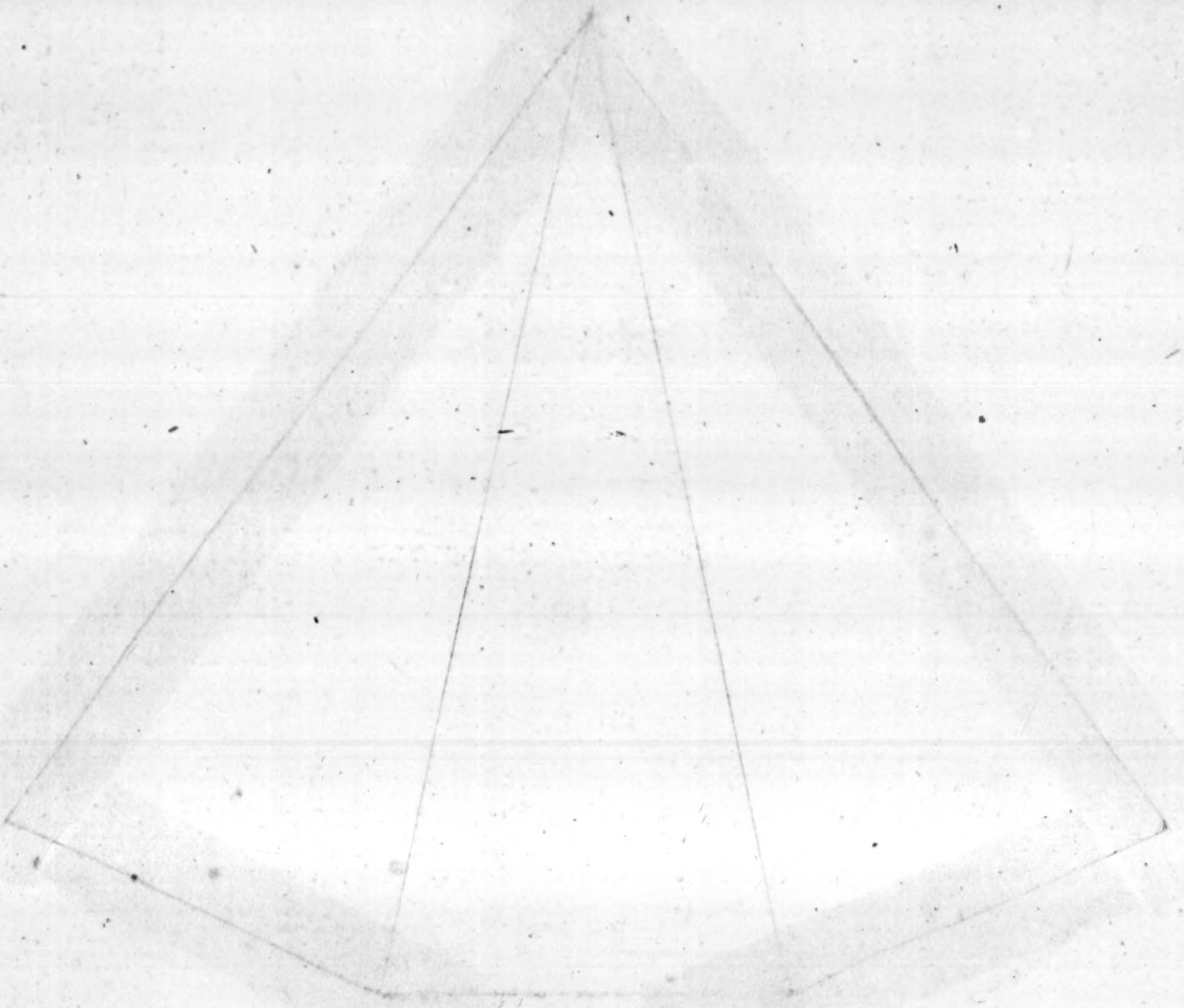


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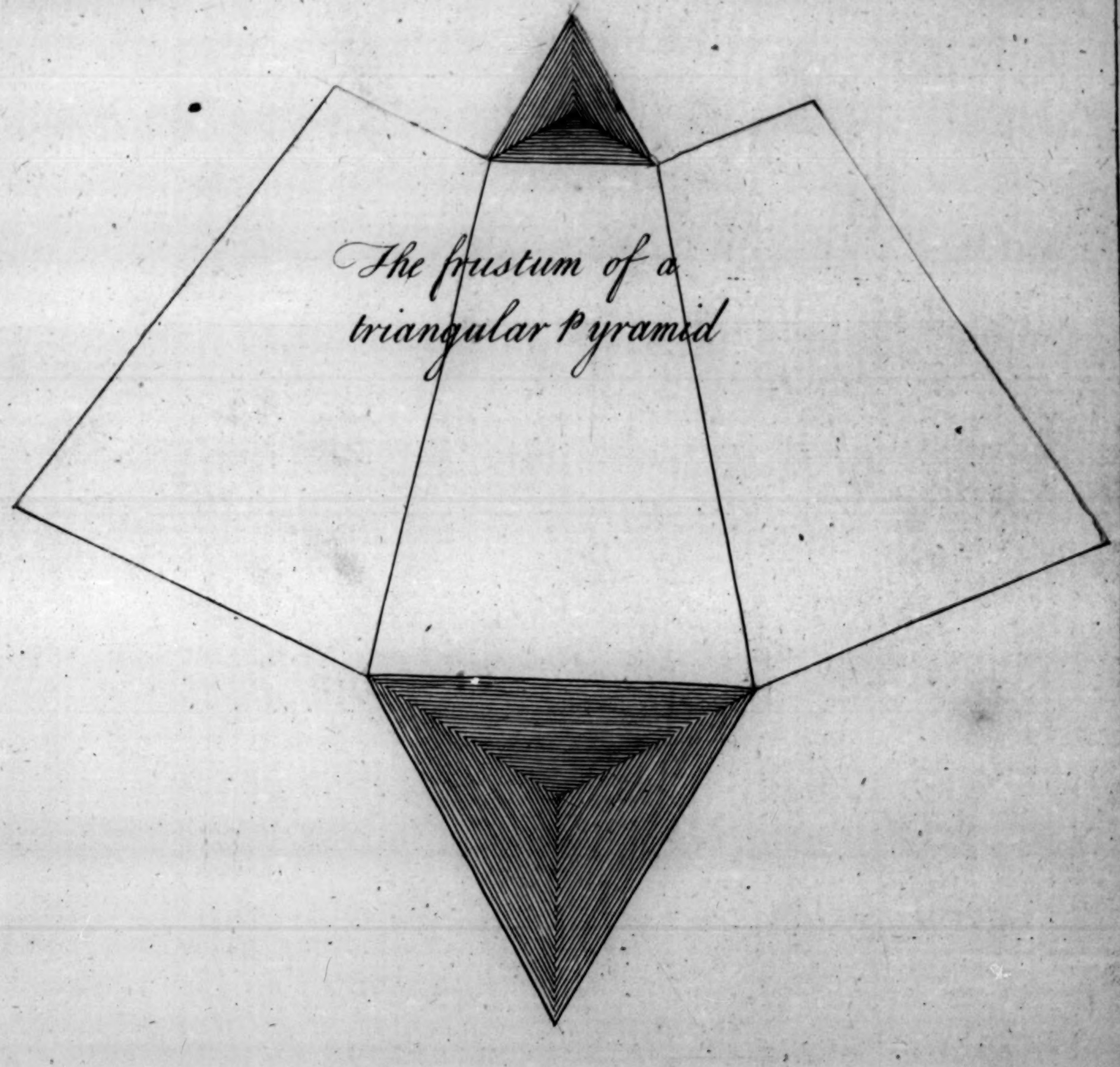
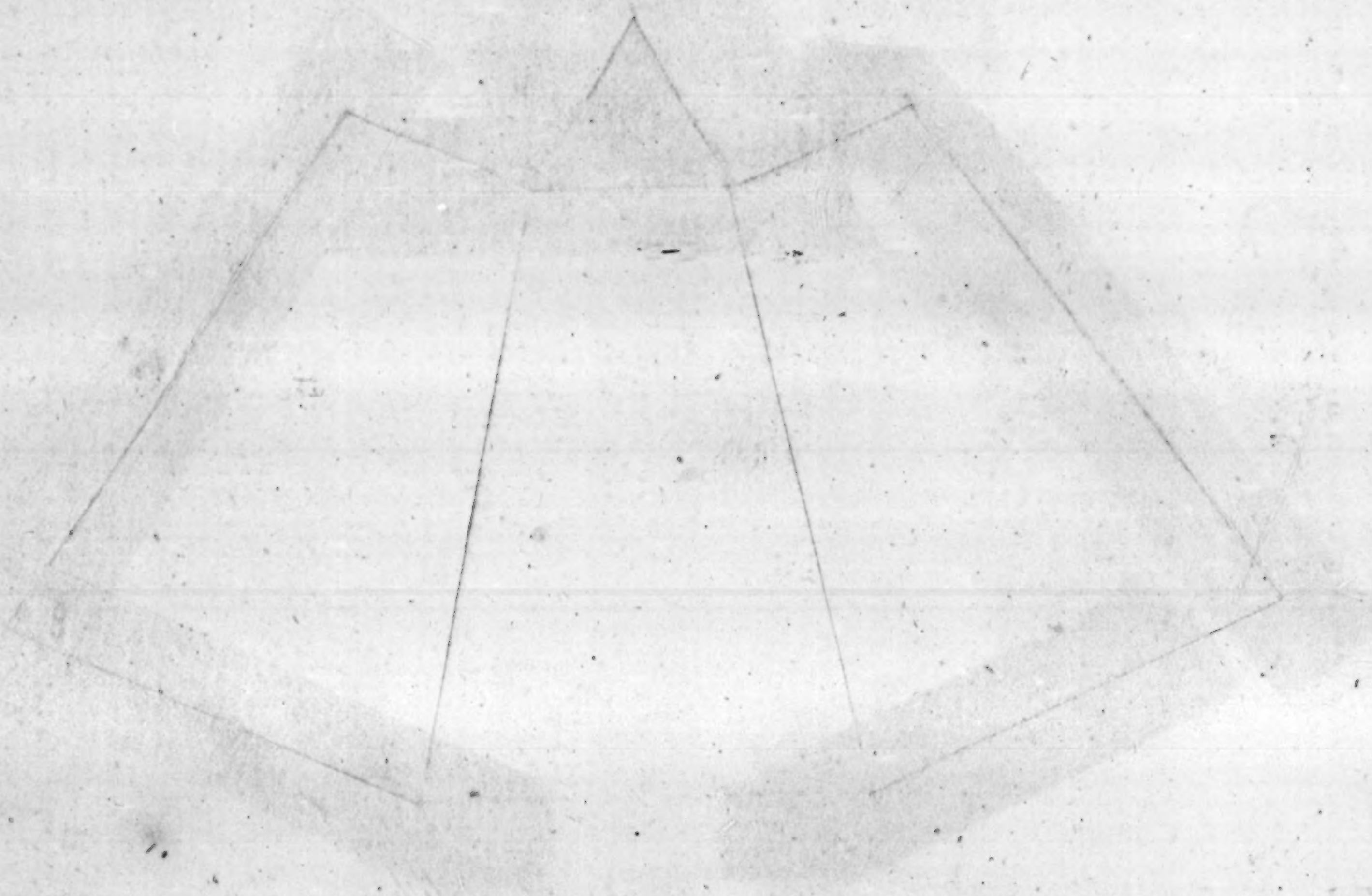
Plate XXV

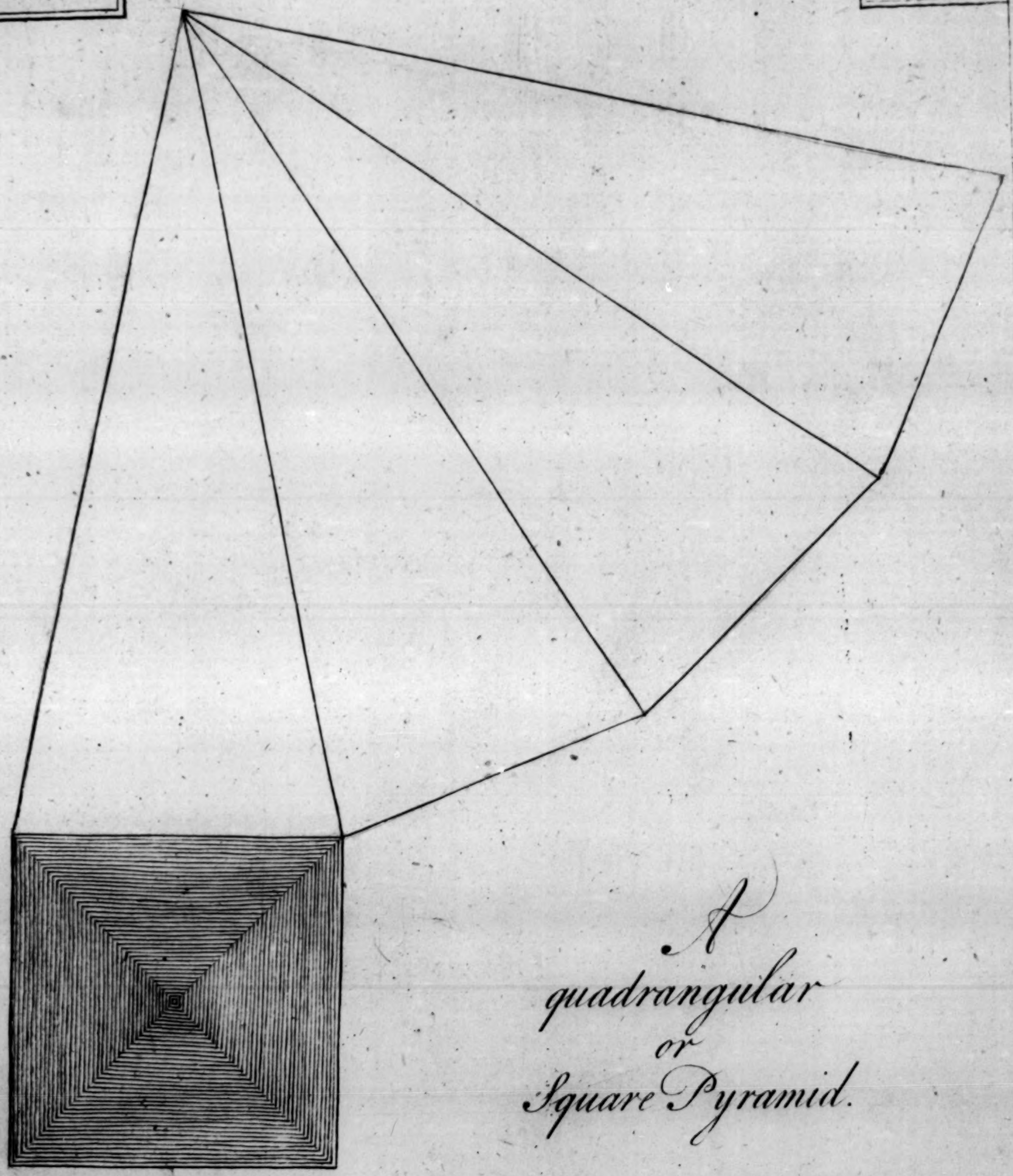
Book. 5.





Book. 5.

Plate XXVI



A
quadrangular
or
Square Pyramid.

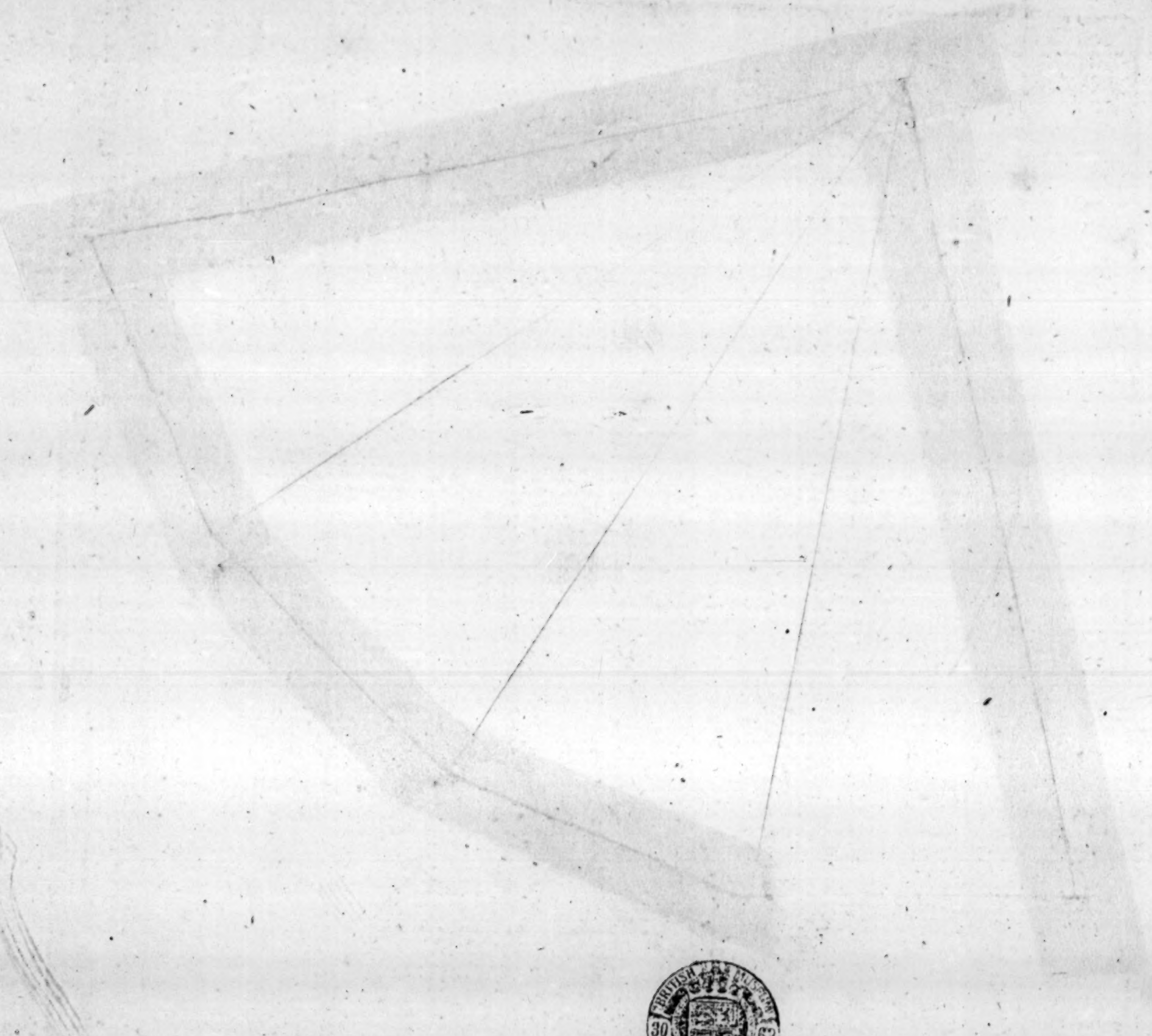
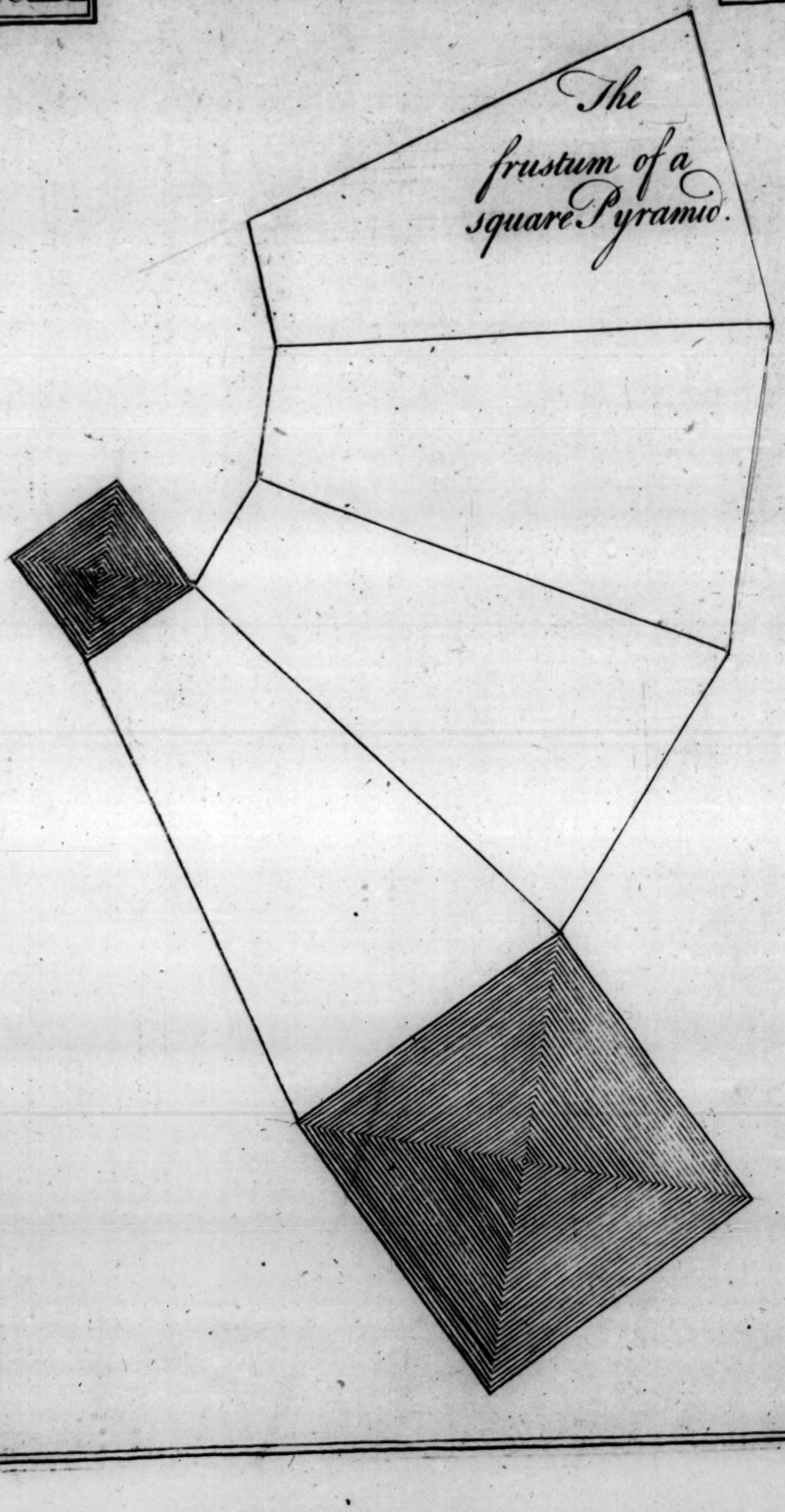
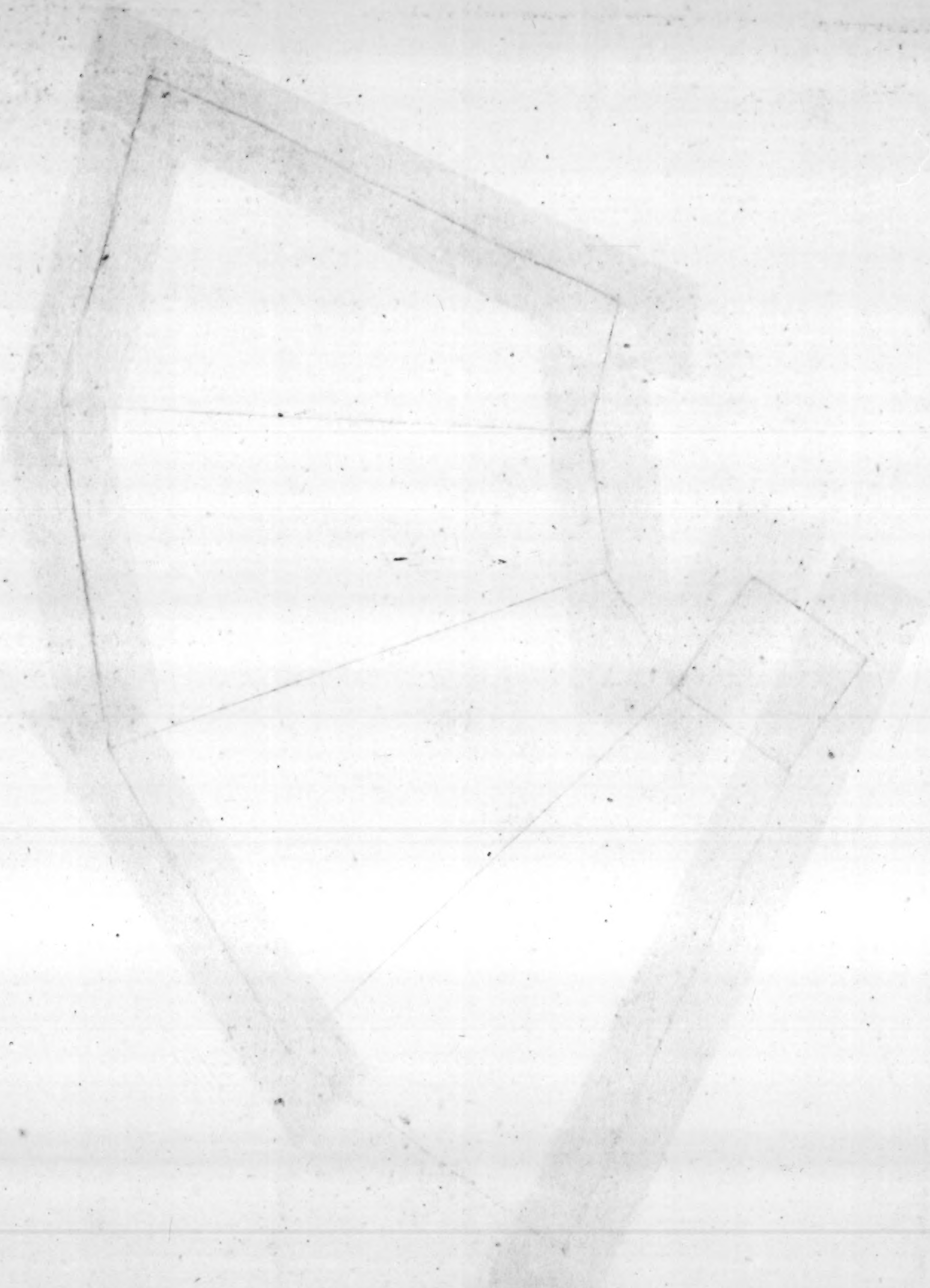


Plate XXVII

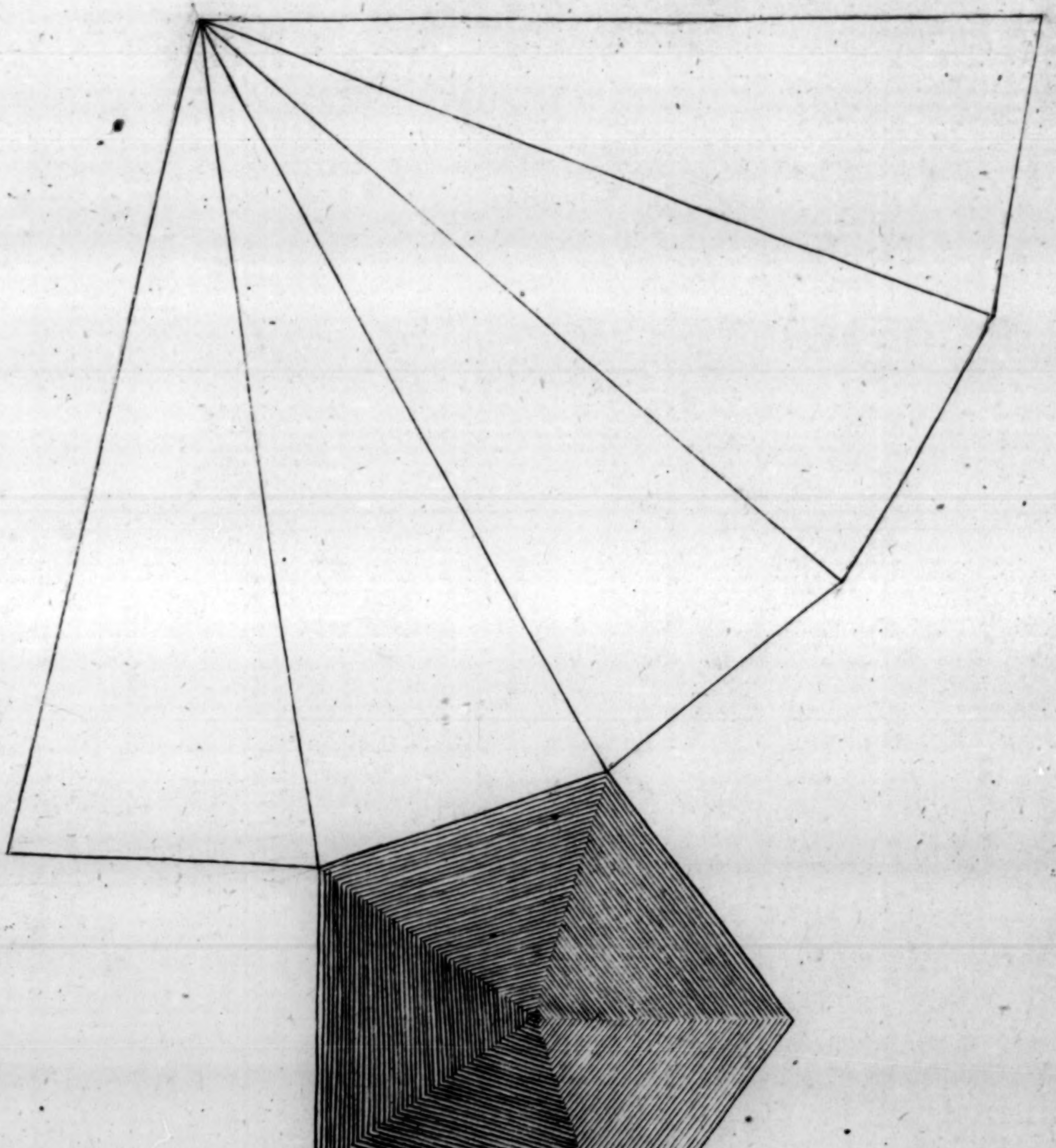
Book 5.

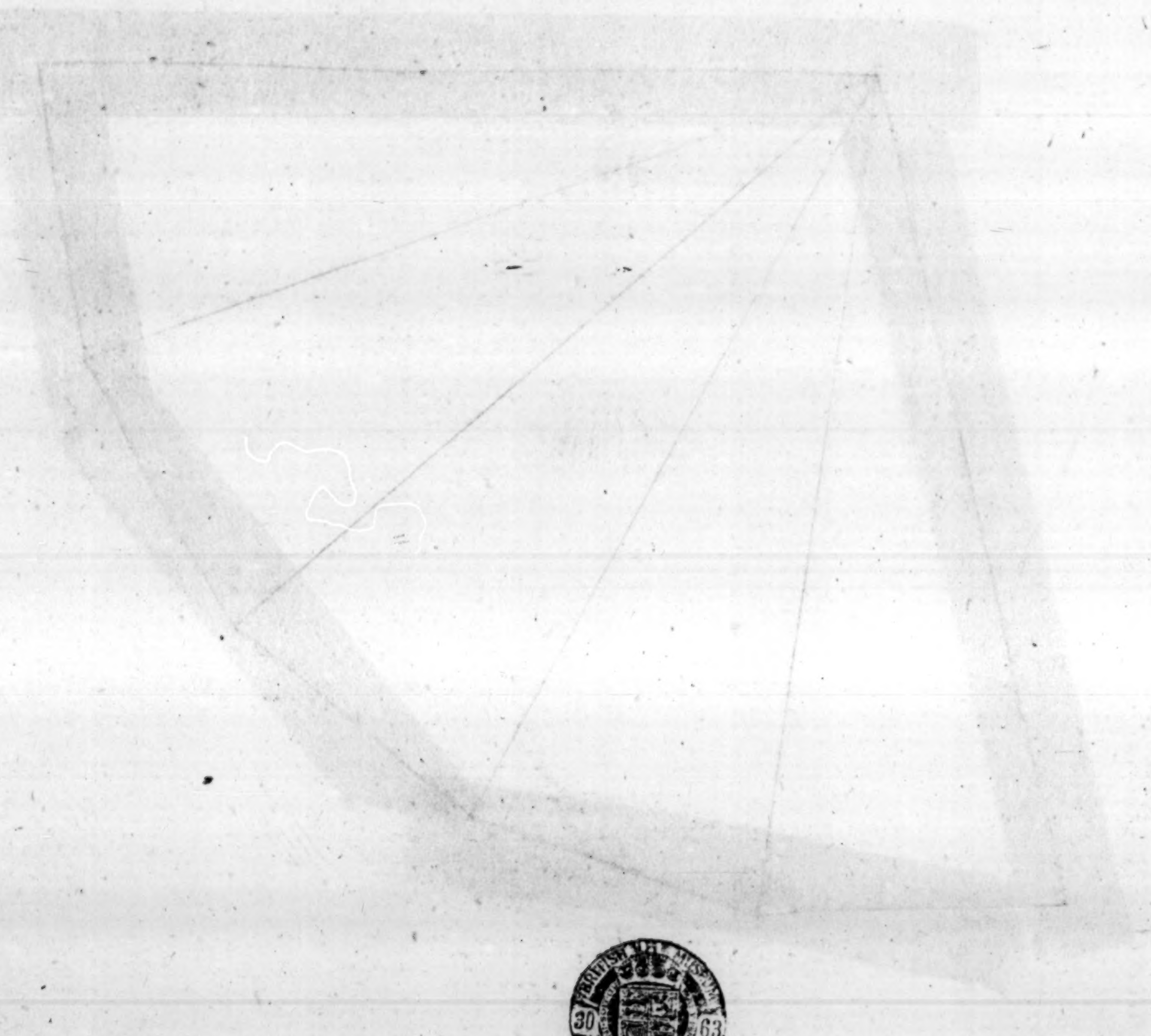


The frustum of a square Pyramid.

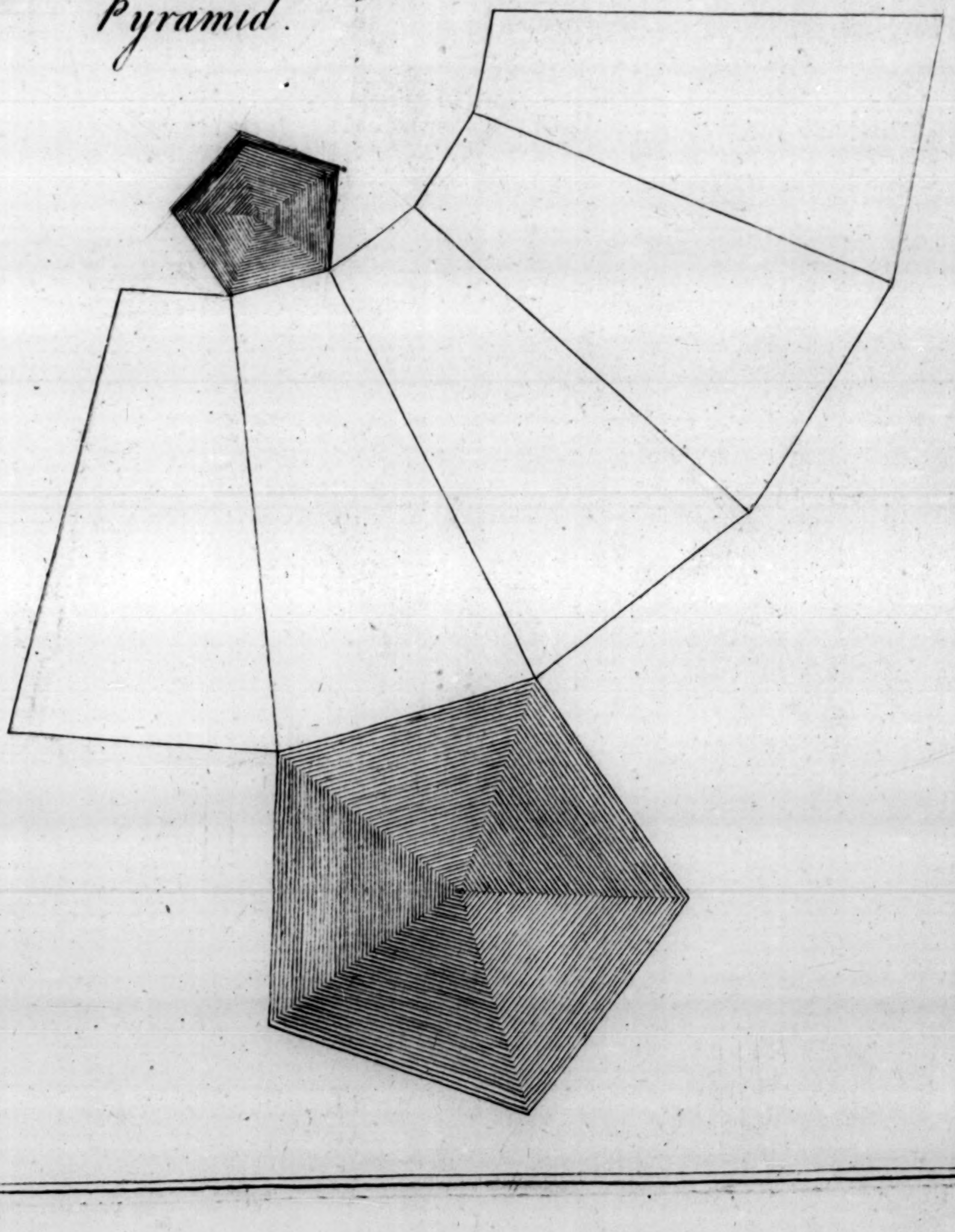


A Pentagonal Pyramid.





The frustum of a Pentagonal Pyramid.



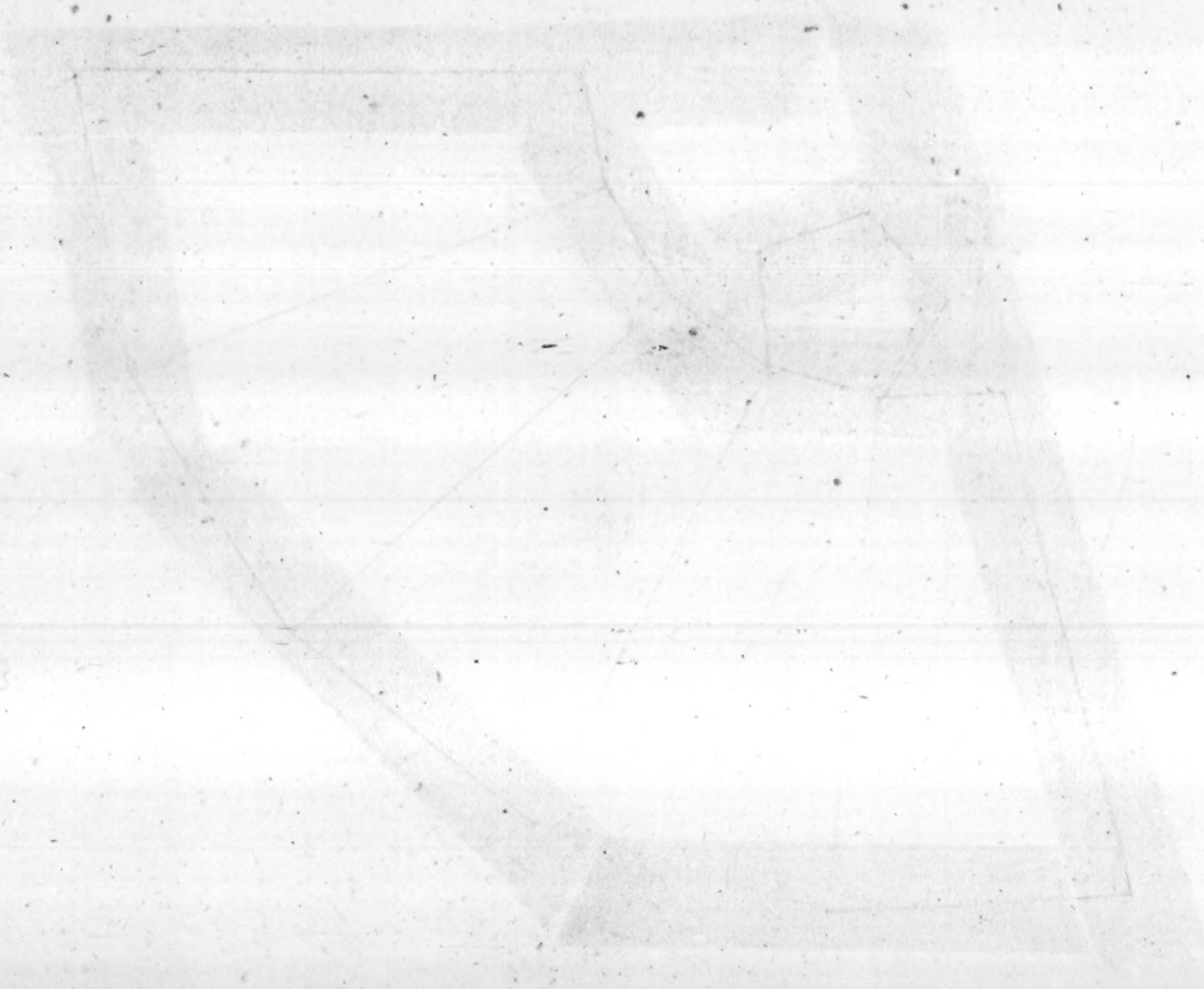


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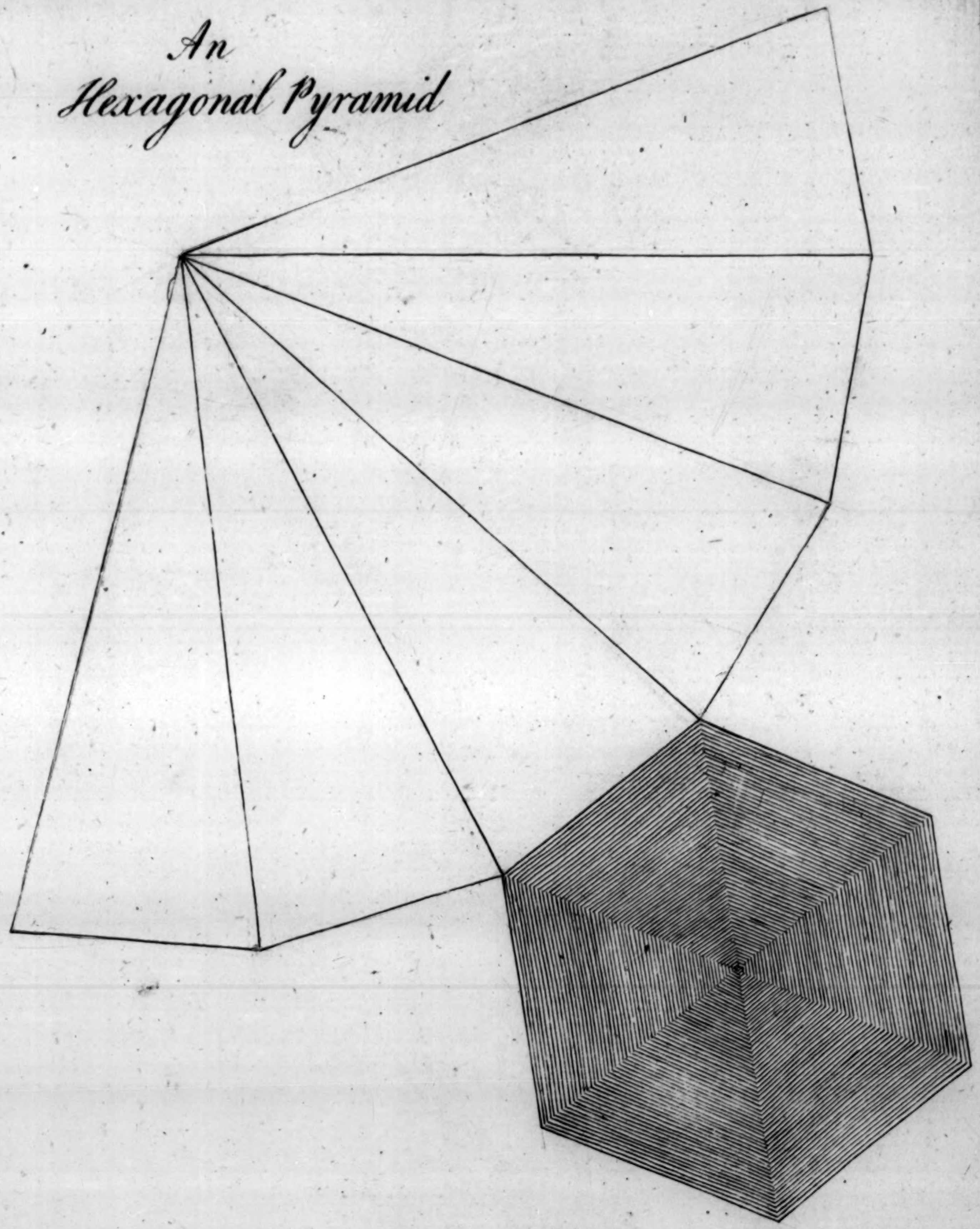
Plate XXX

Book. 5.

An Hexagonal Pyramid



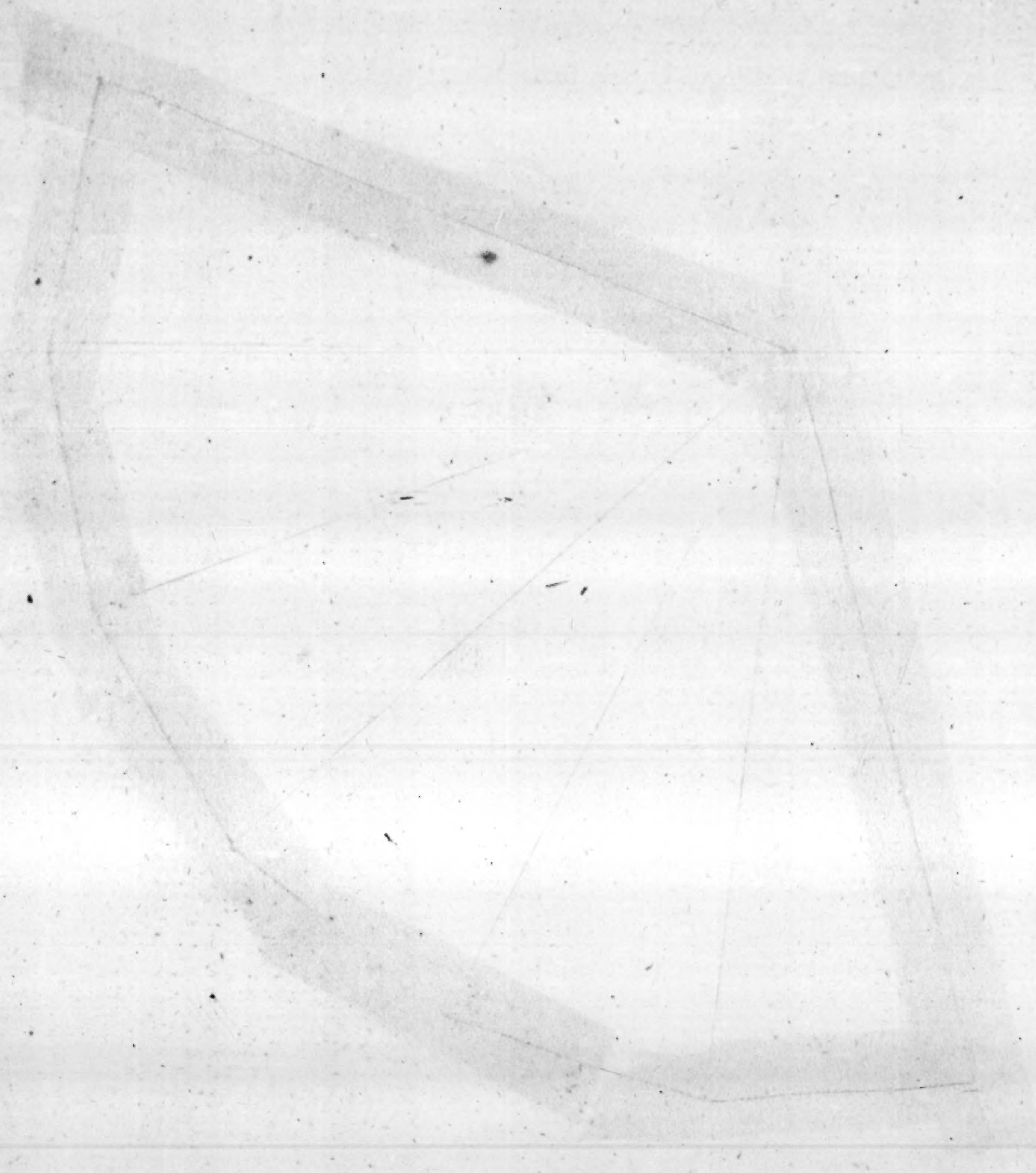


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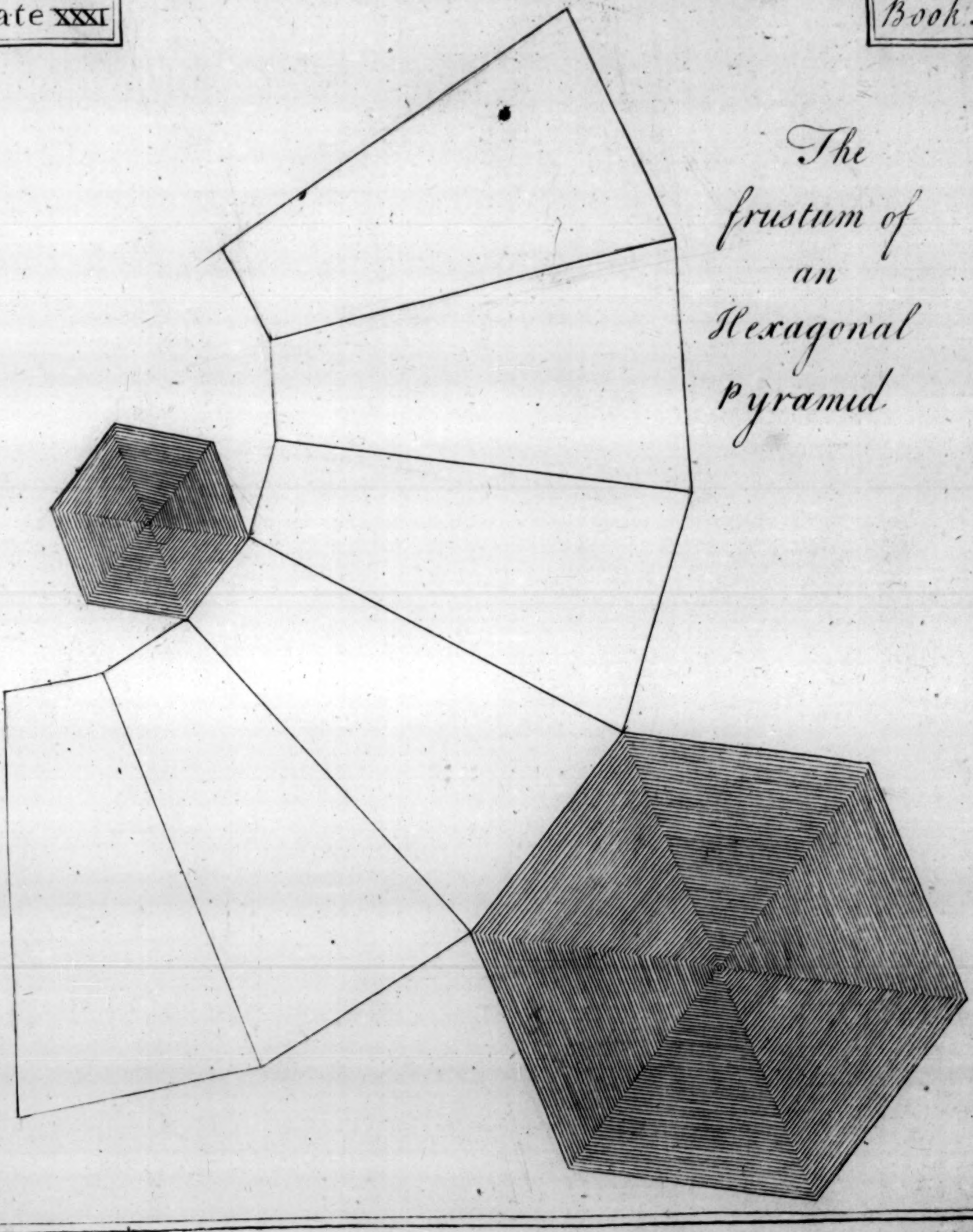
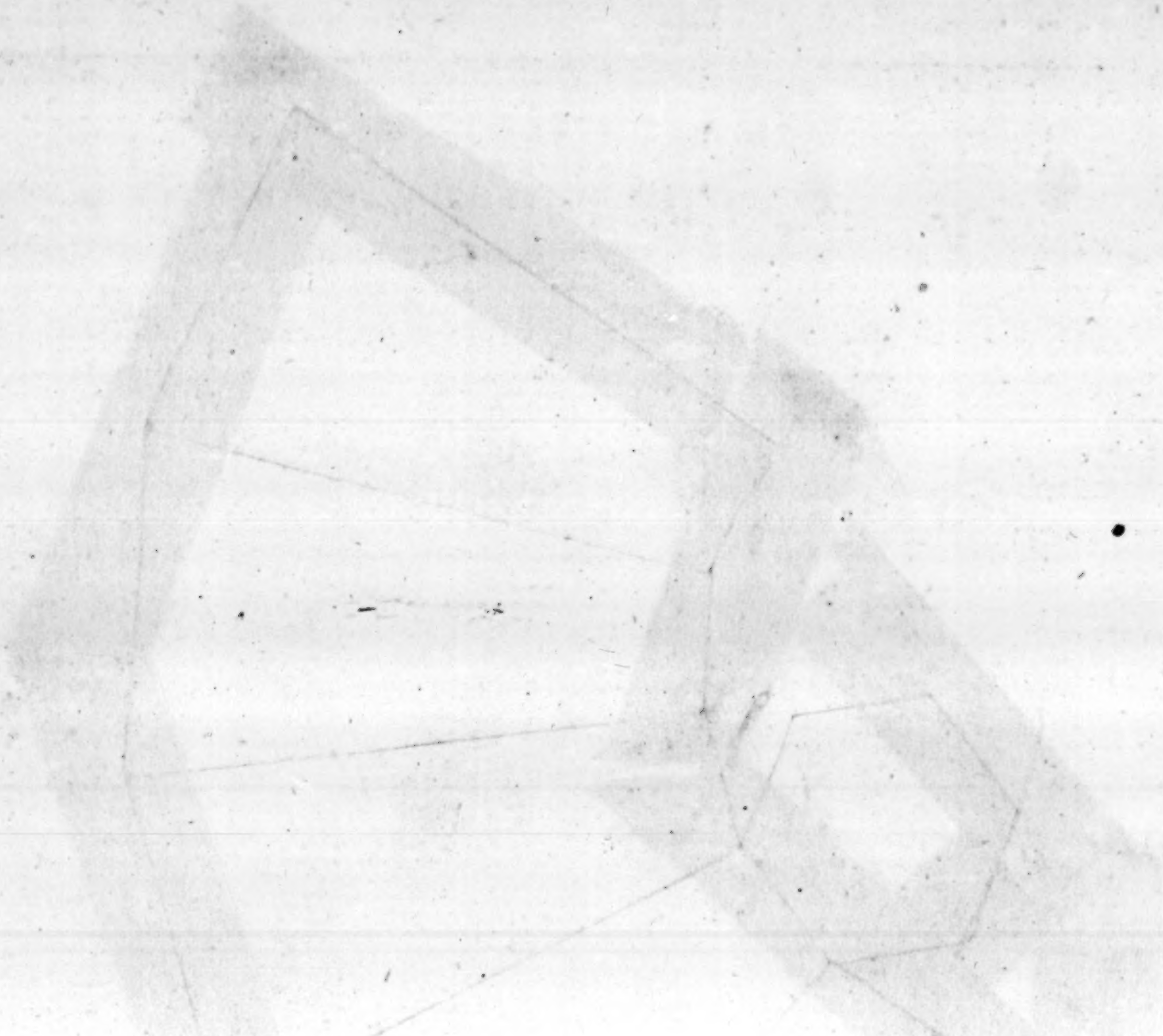
Plate XXXI

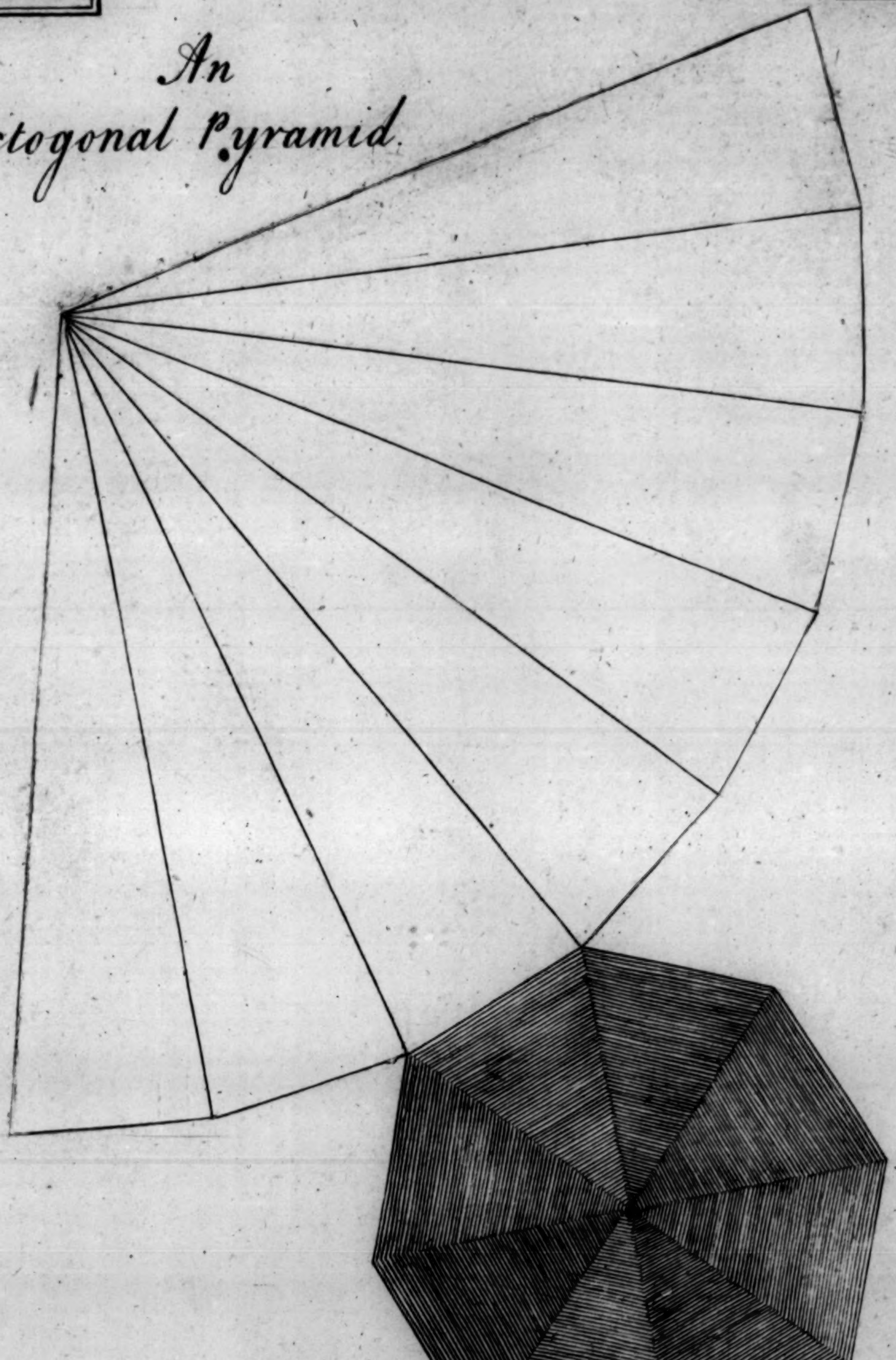
Book 5.



The frustum of an Hexagonal Pyramid



An
Octogonal Pyramid.



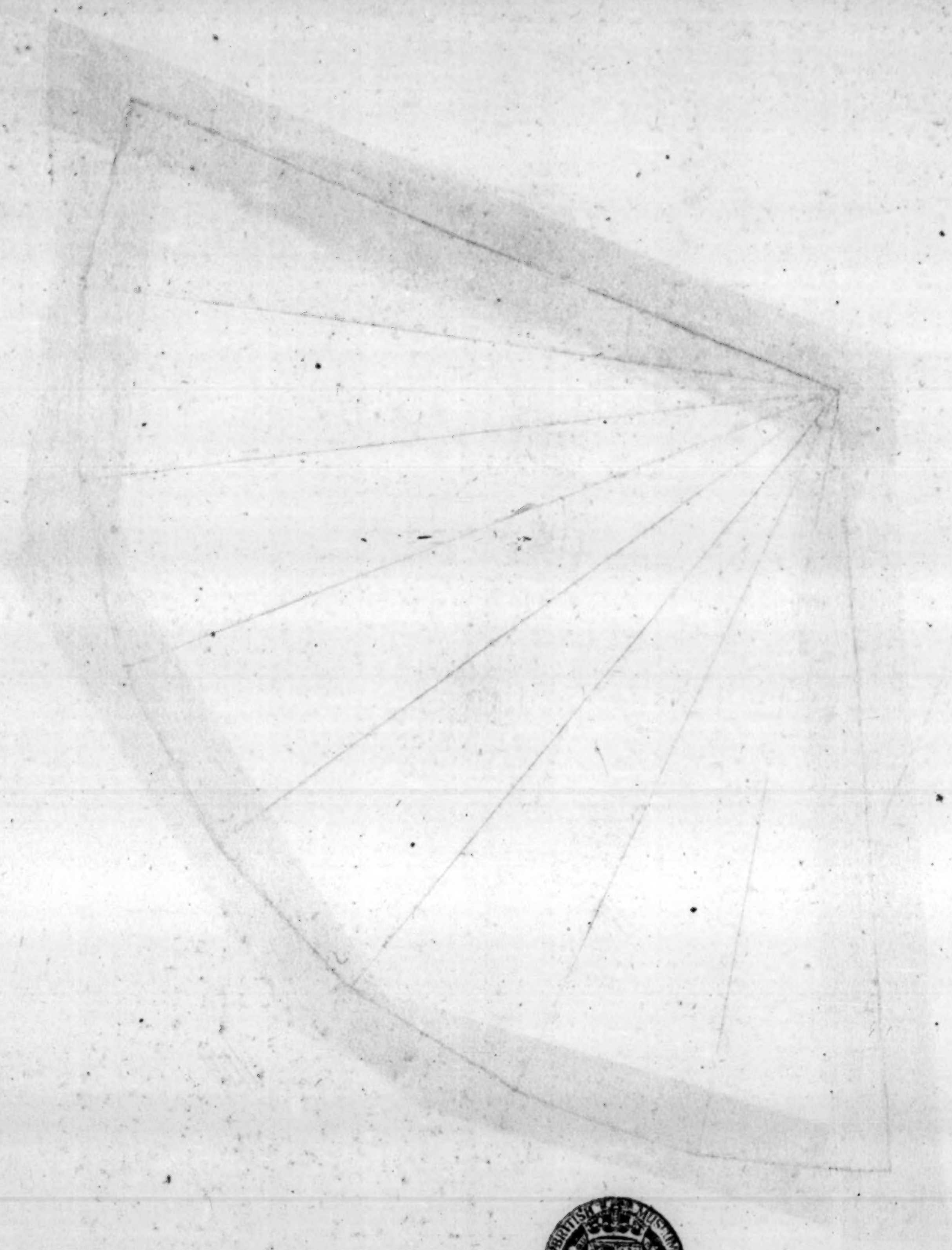


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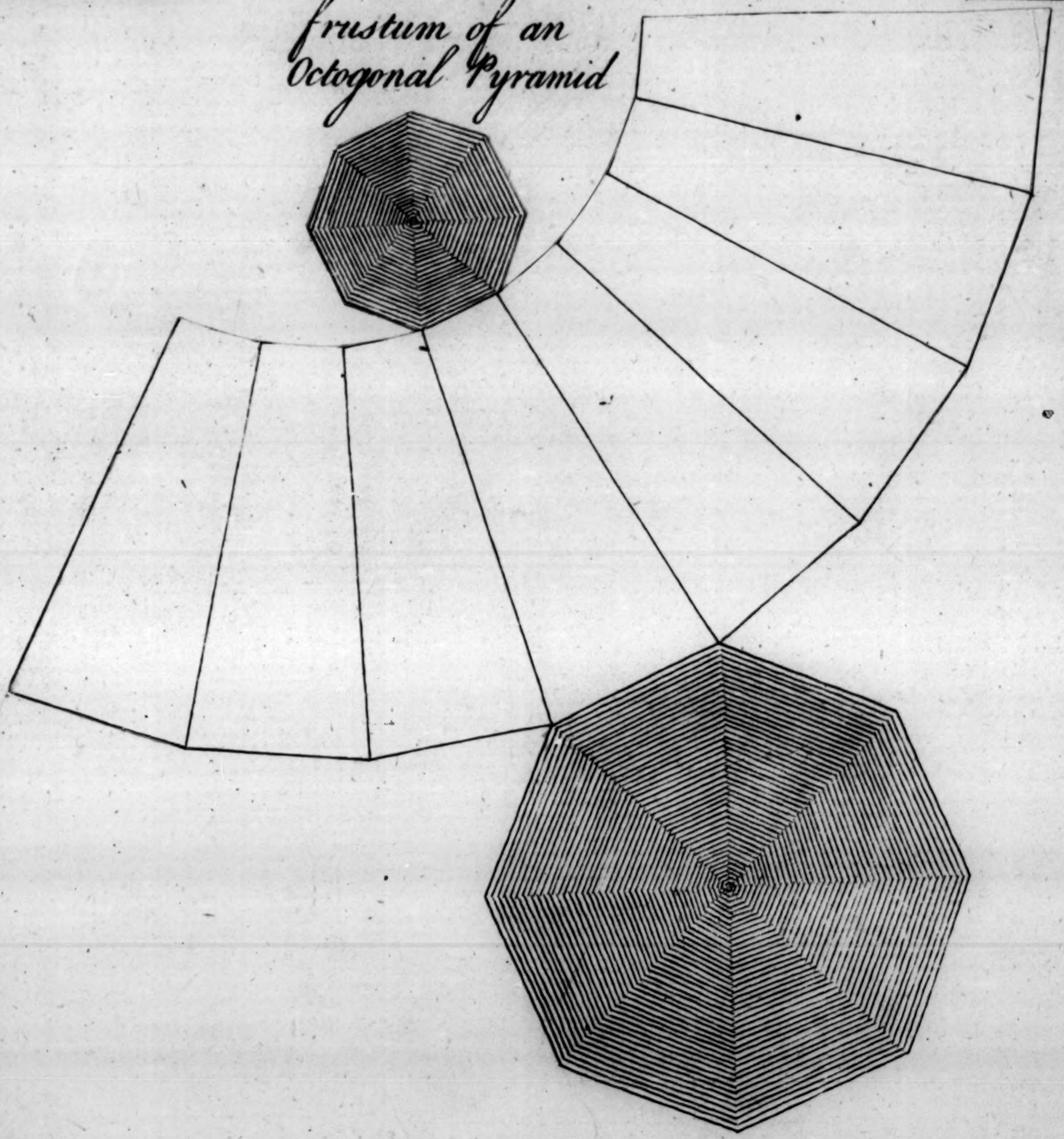
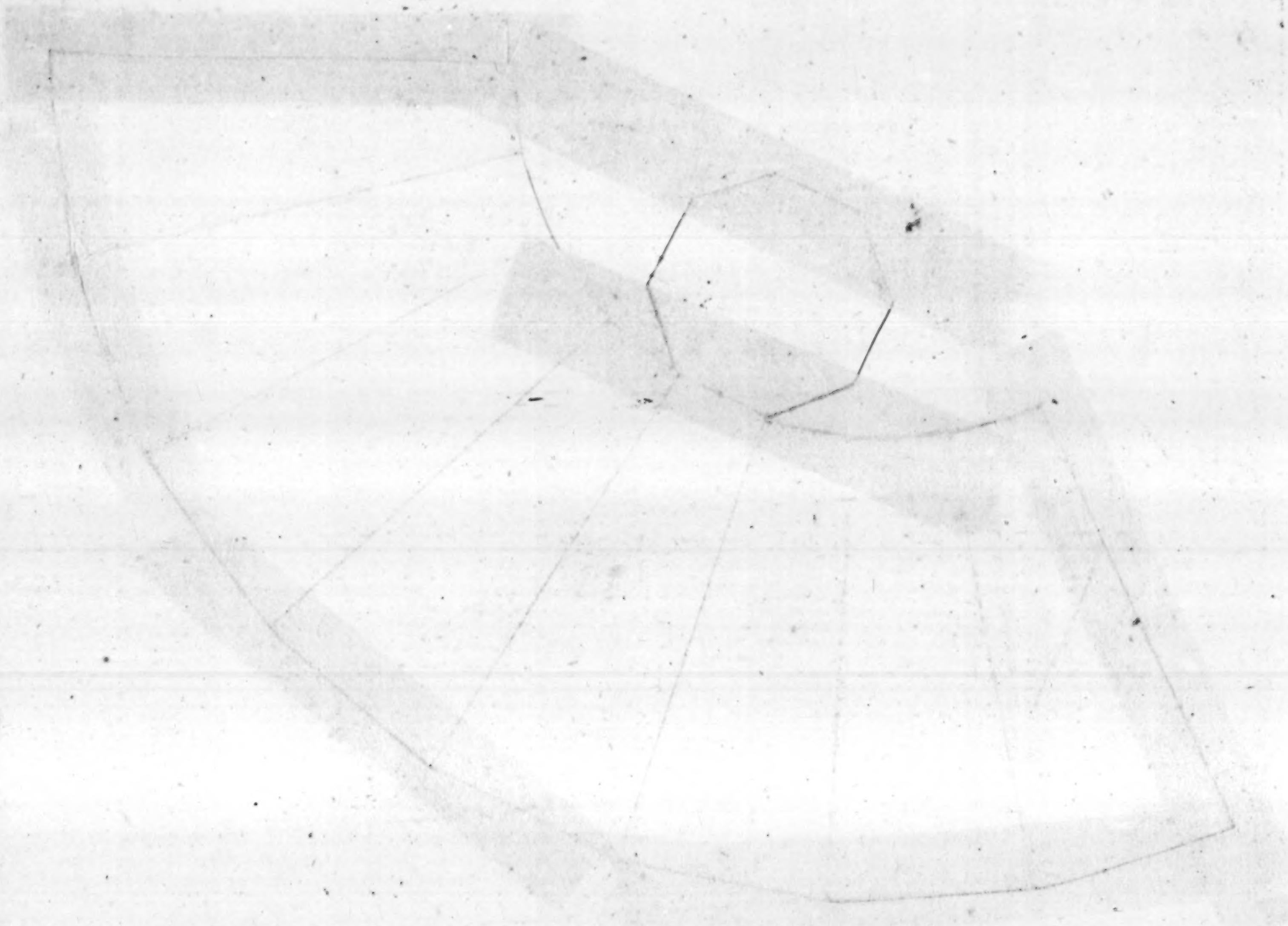
Plate XXXIII

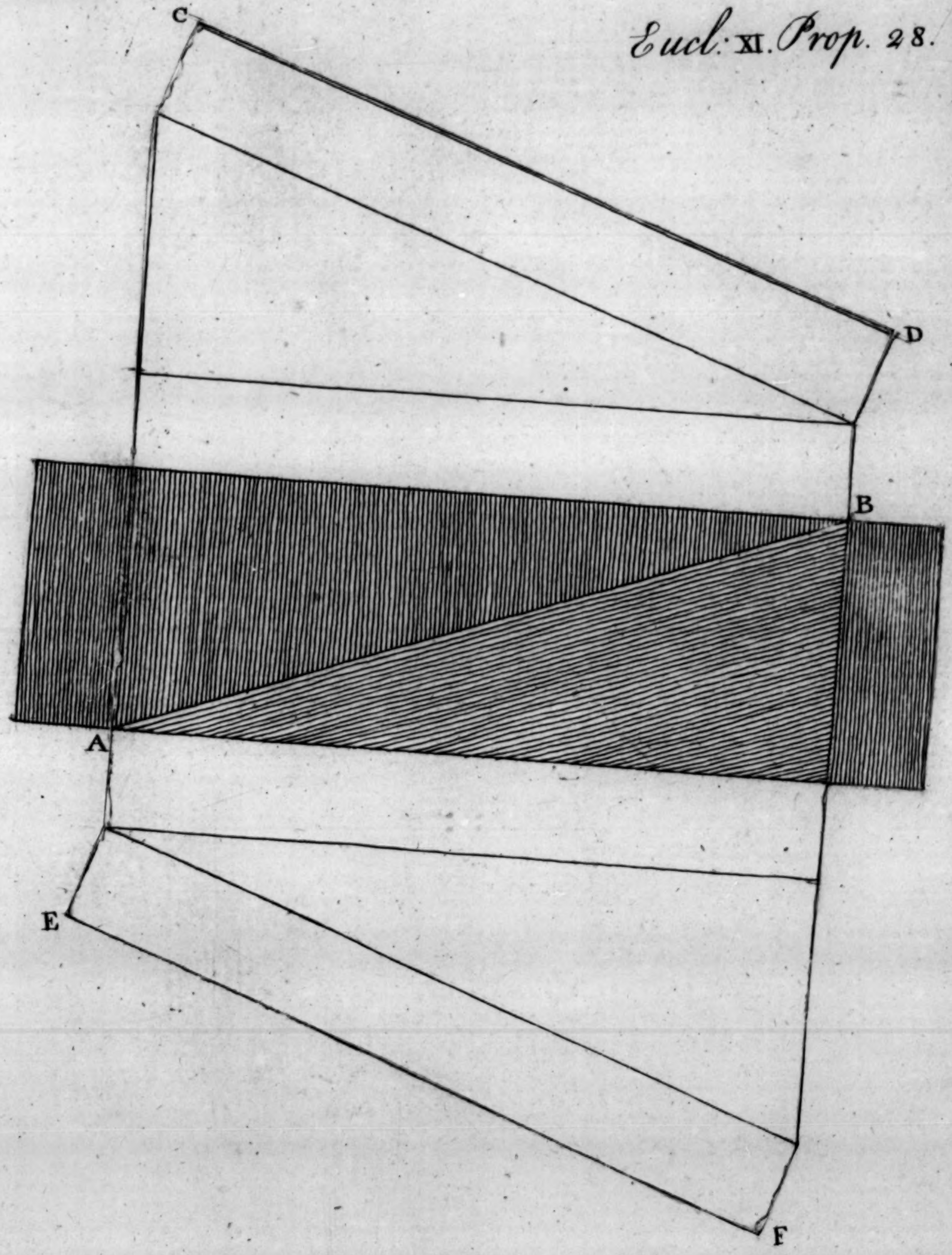
Book. 5.

The frustum of an Octogonal Pyramid





Eucl: XI. Prop. 28.



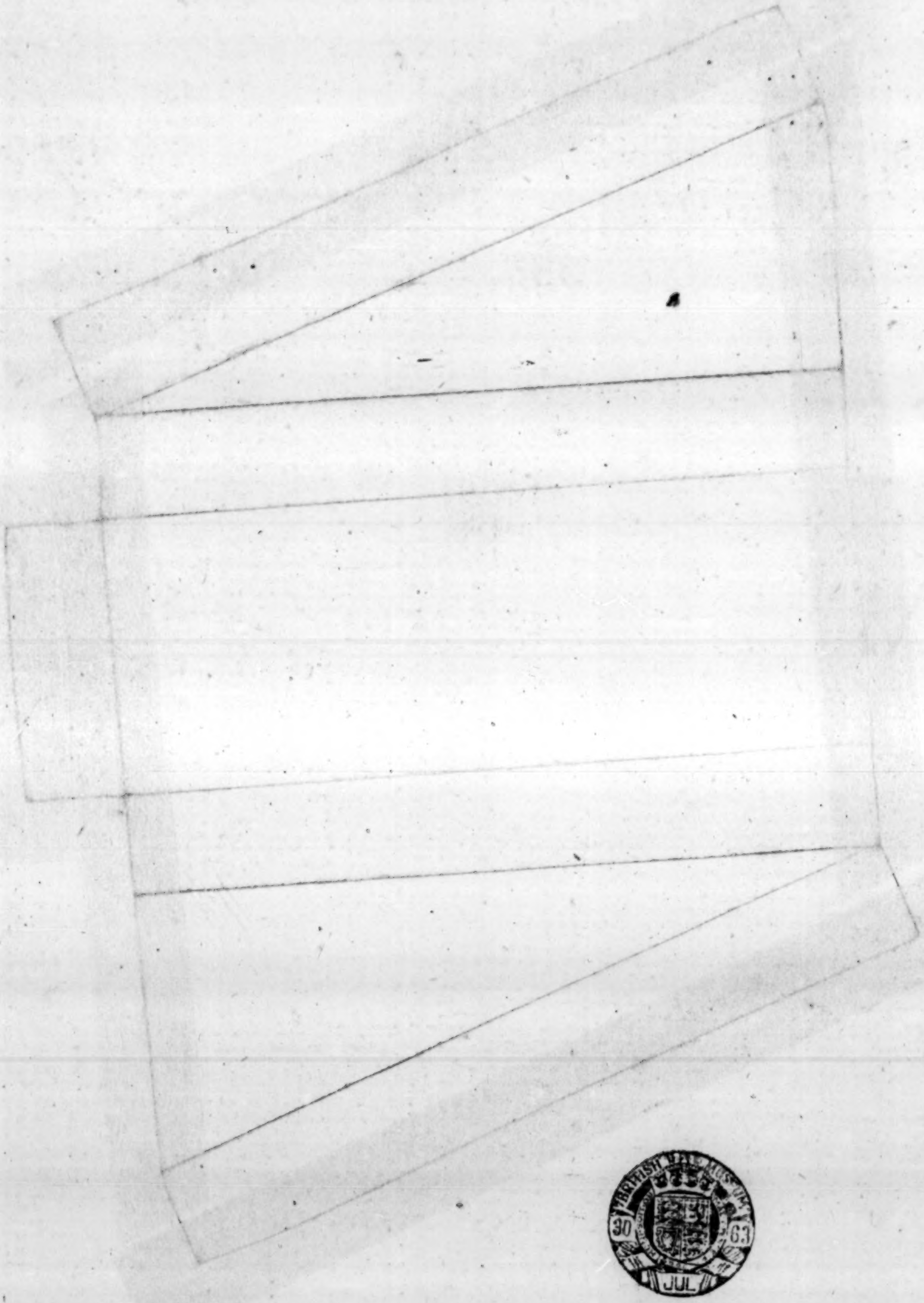
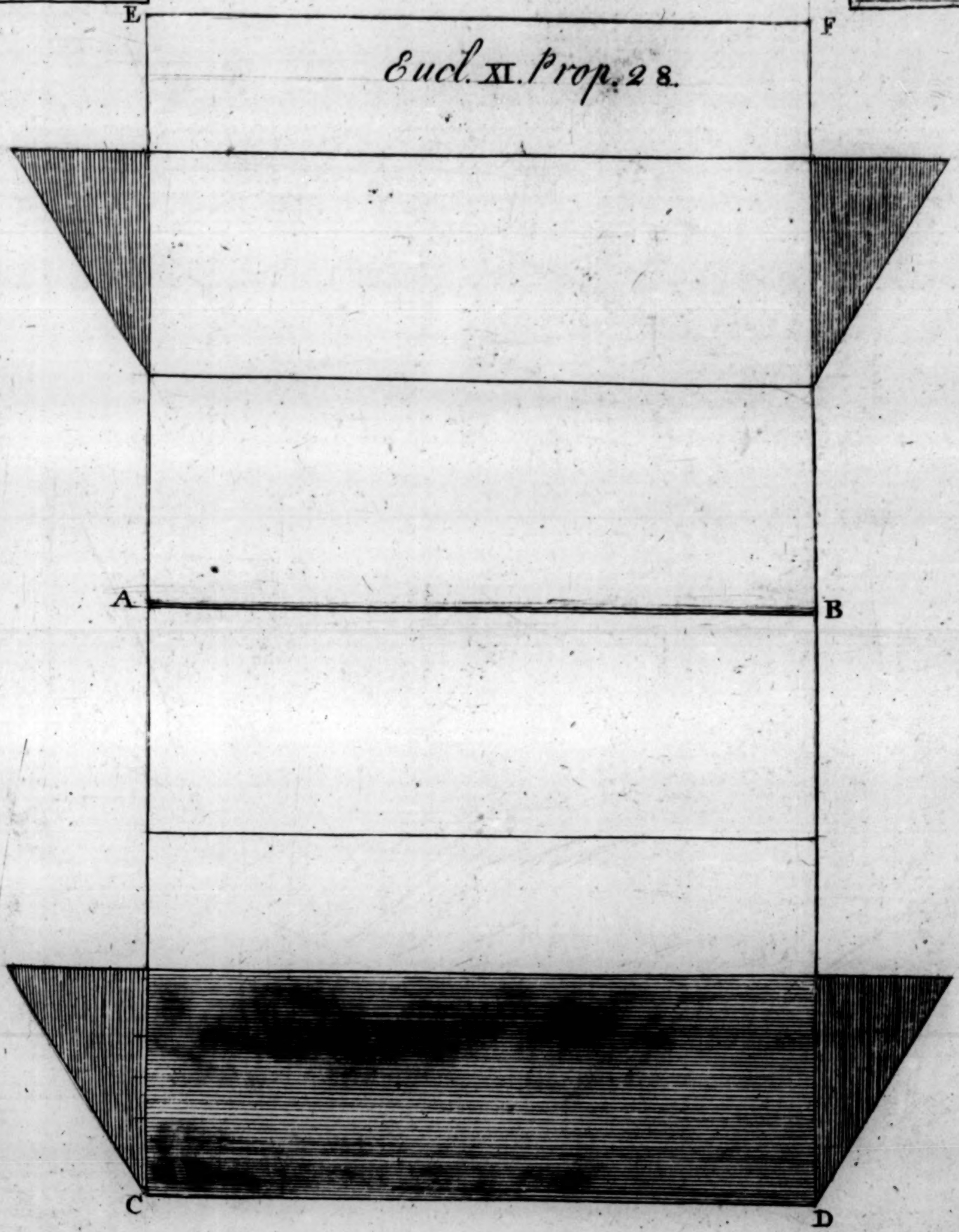


Plate XXXV

Book 6


Eucl. XI. Prop. 28.

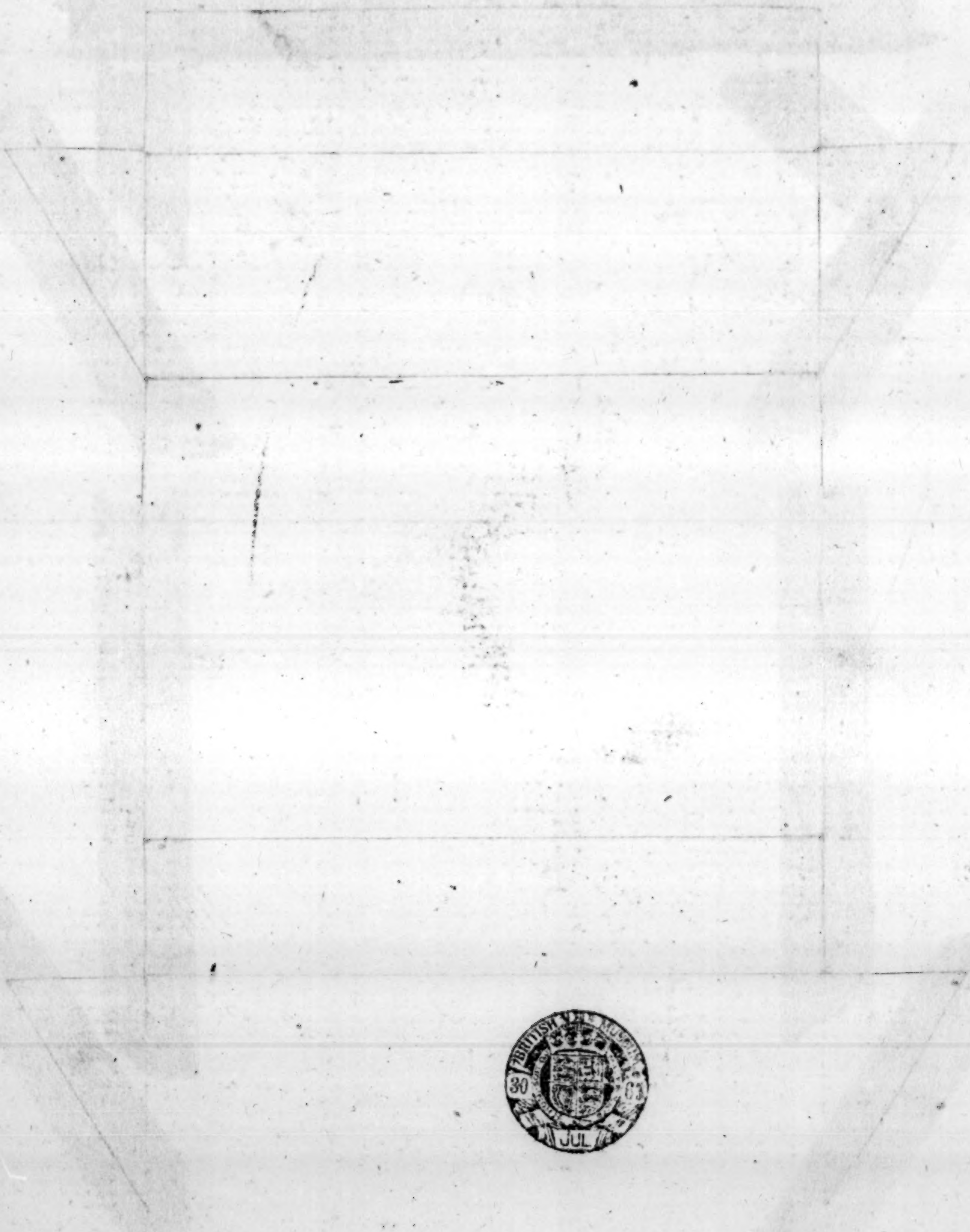
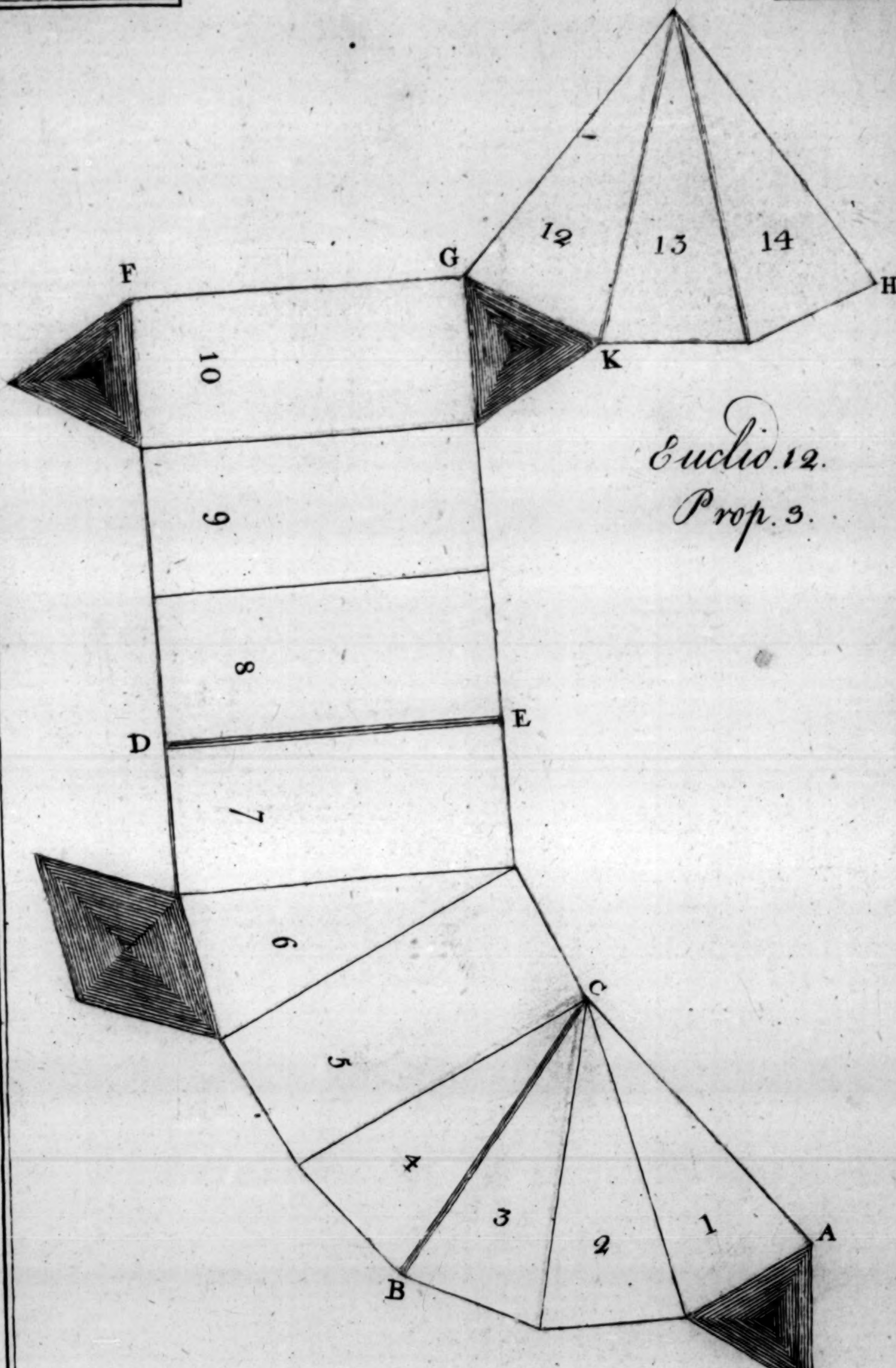


Plate XXXVI

Book. 5.



Euclid.12.
Prop.3.

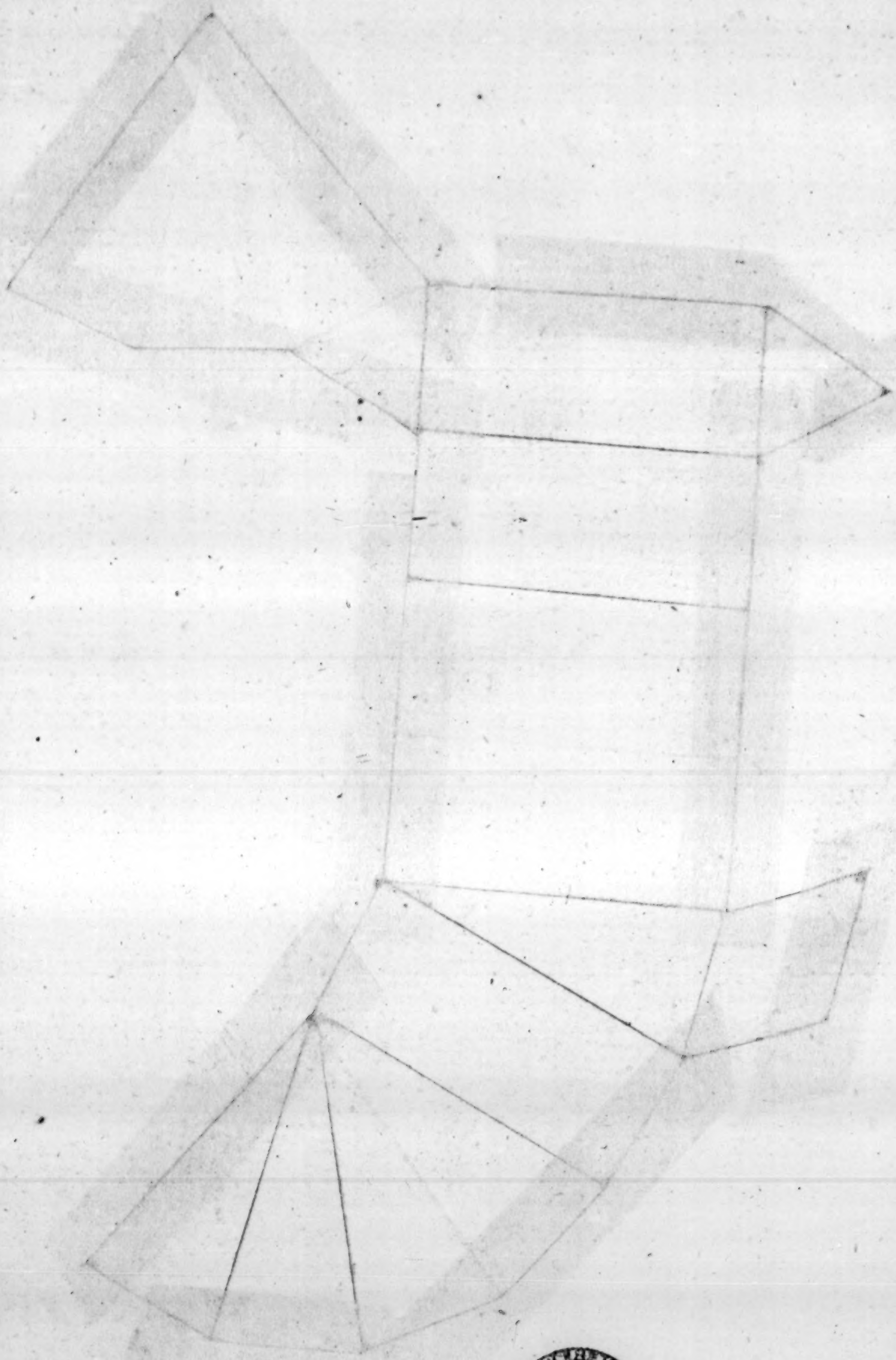
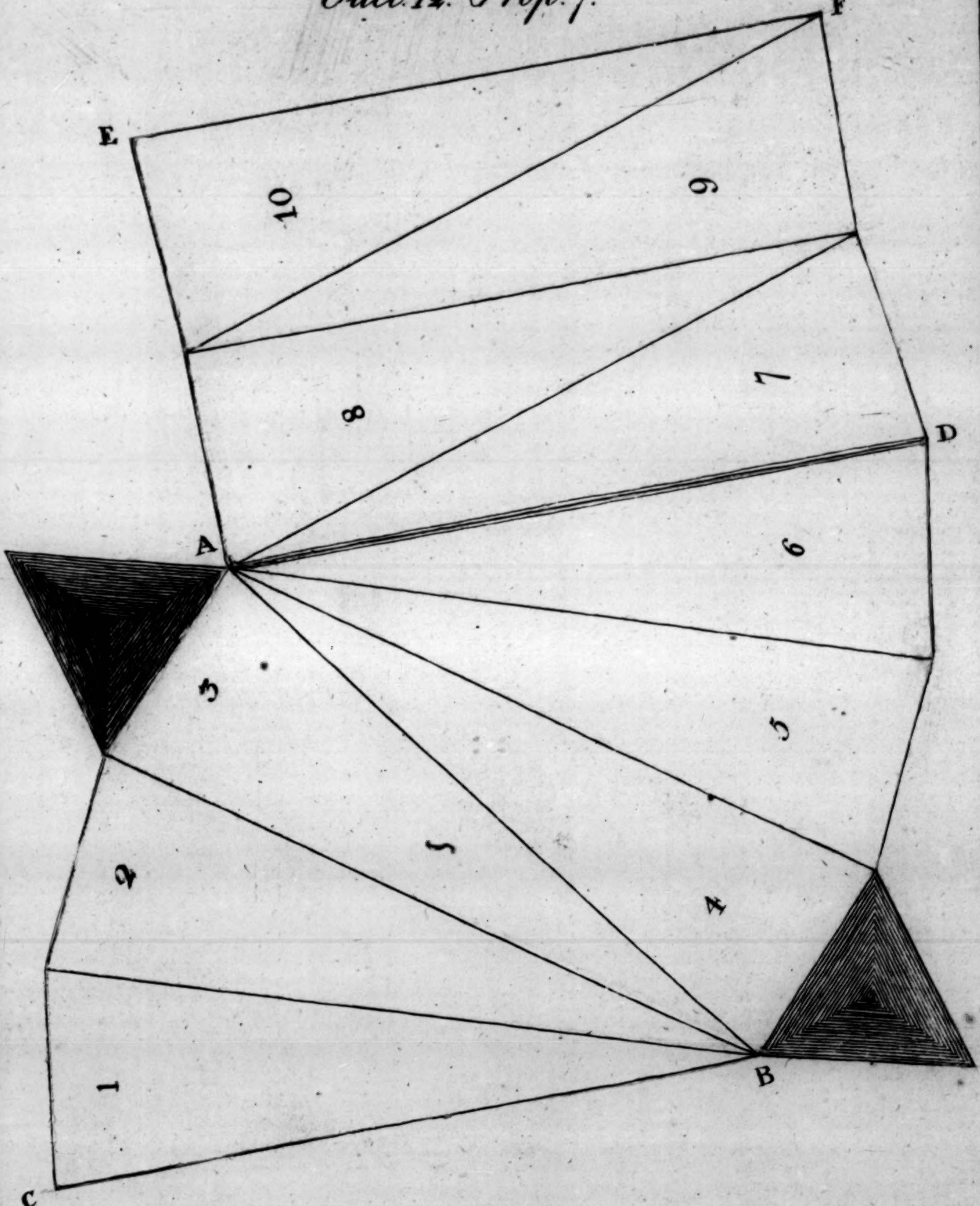
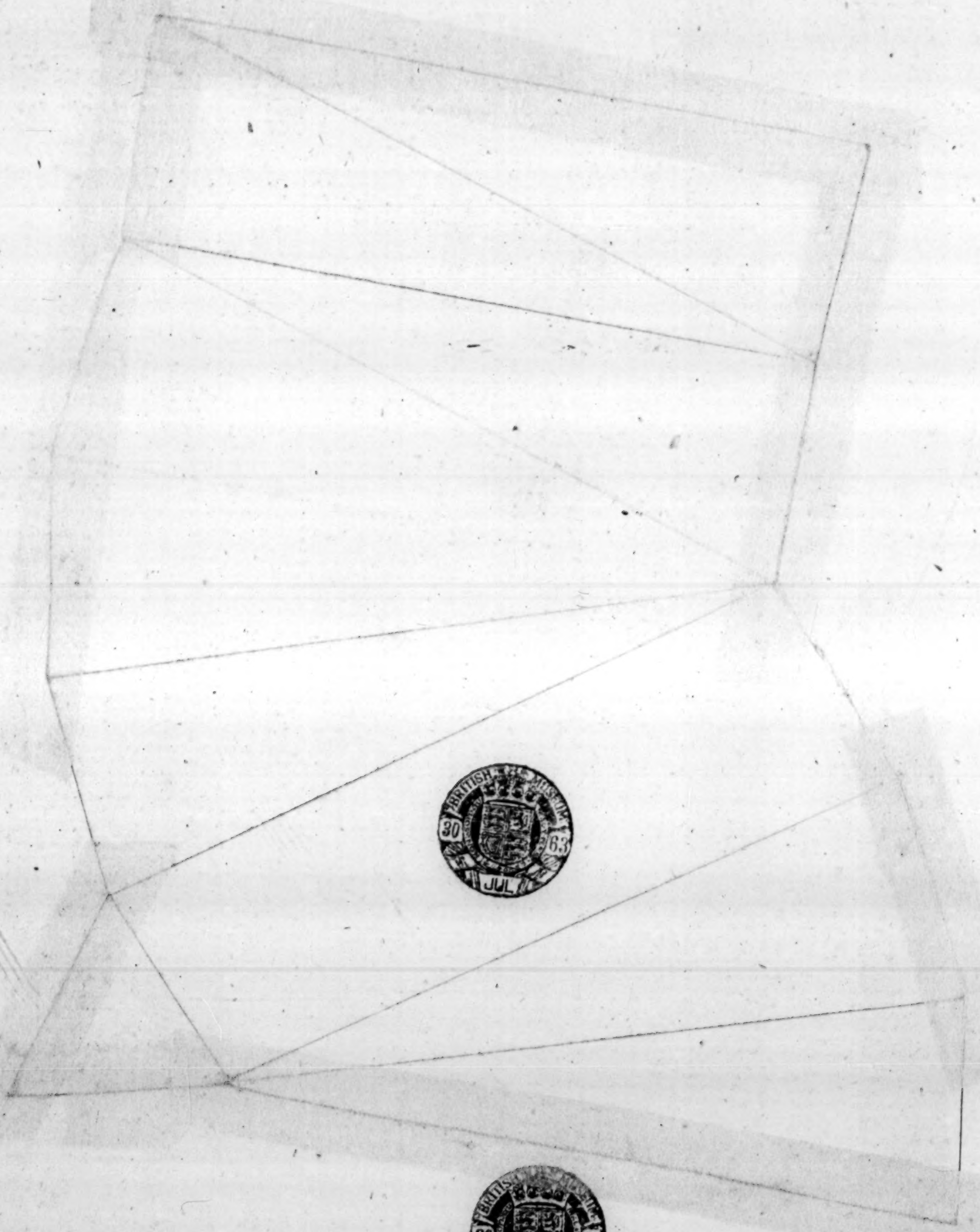


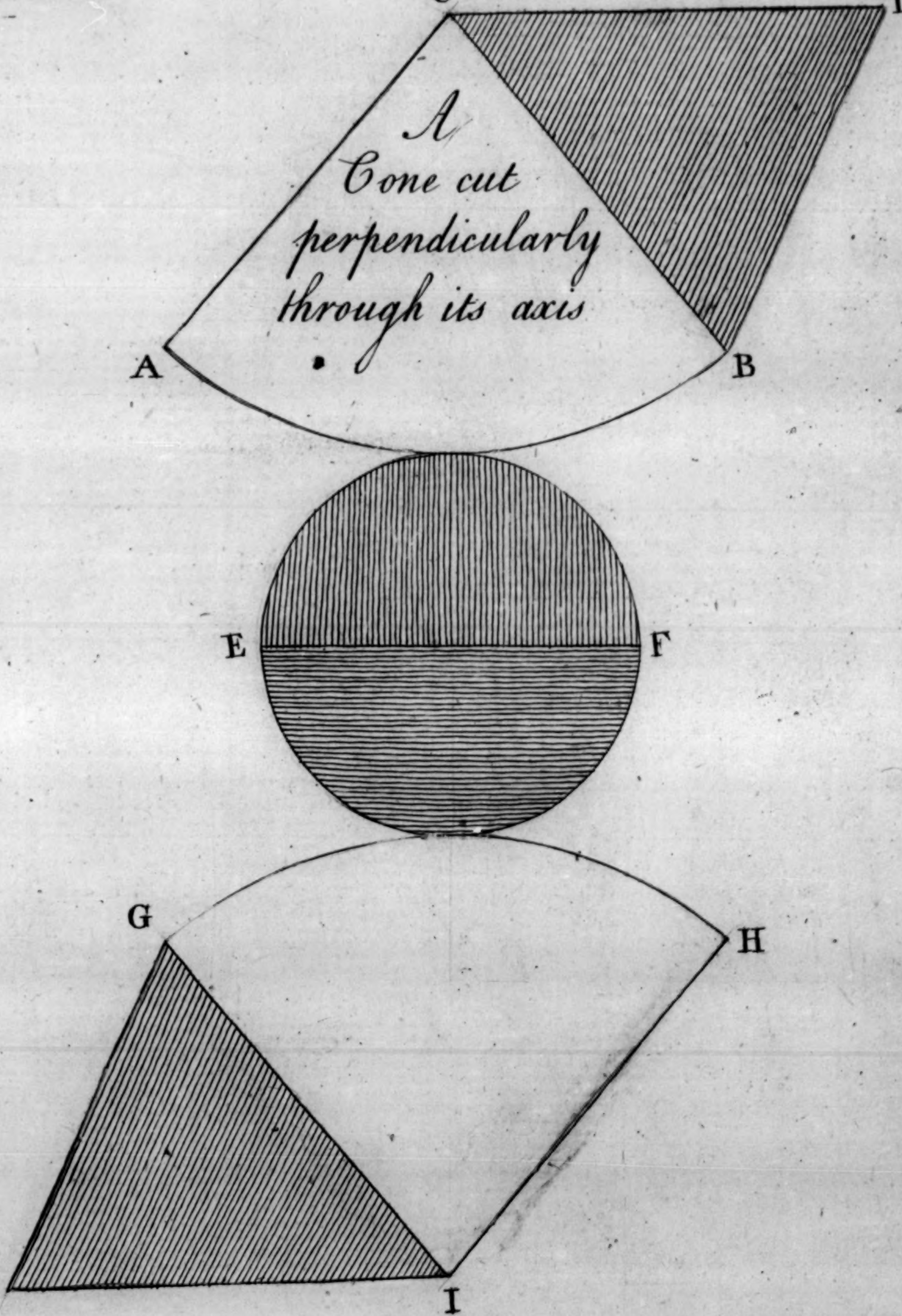
Plate XXXVII

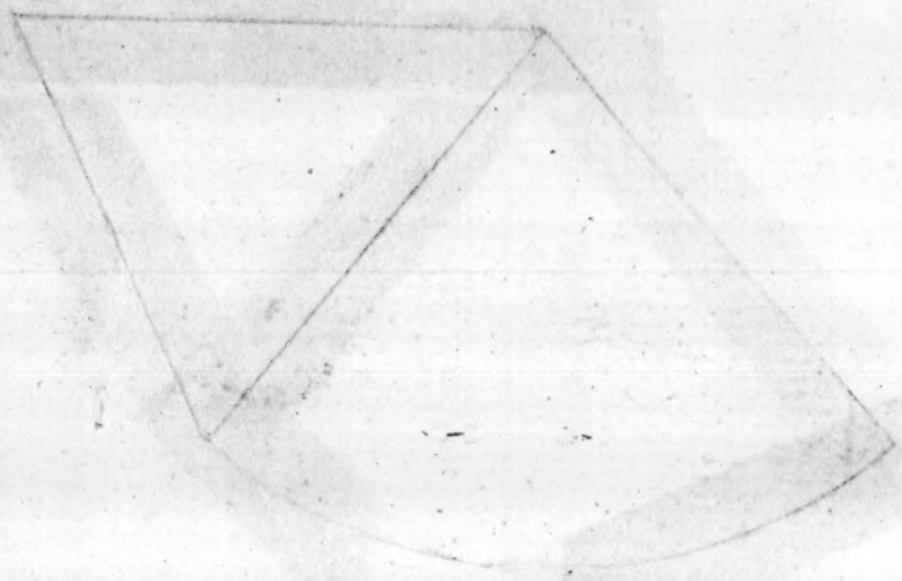
Book.5.

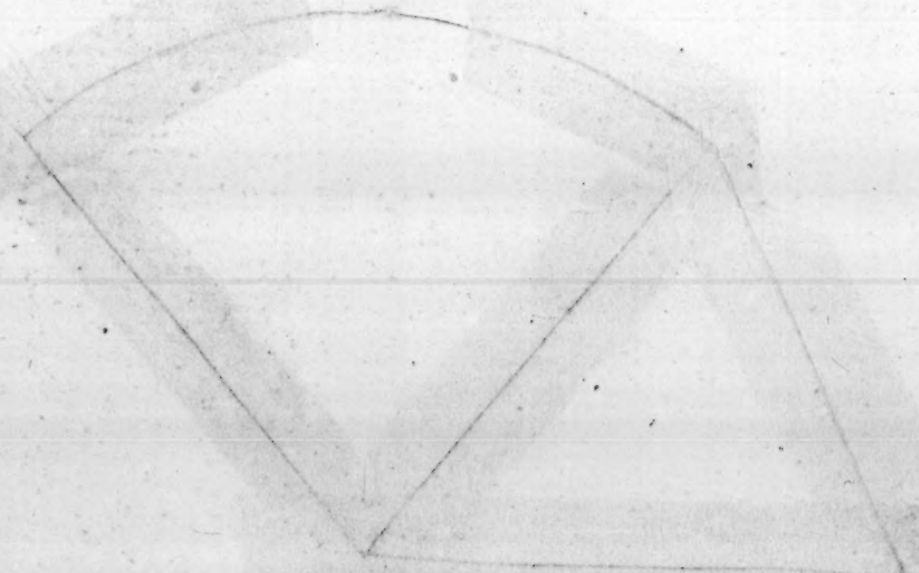
Eucl.12. Prop.7.

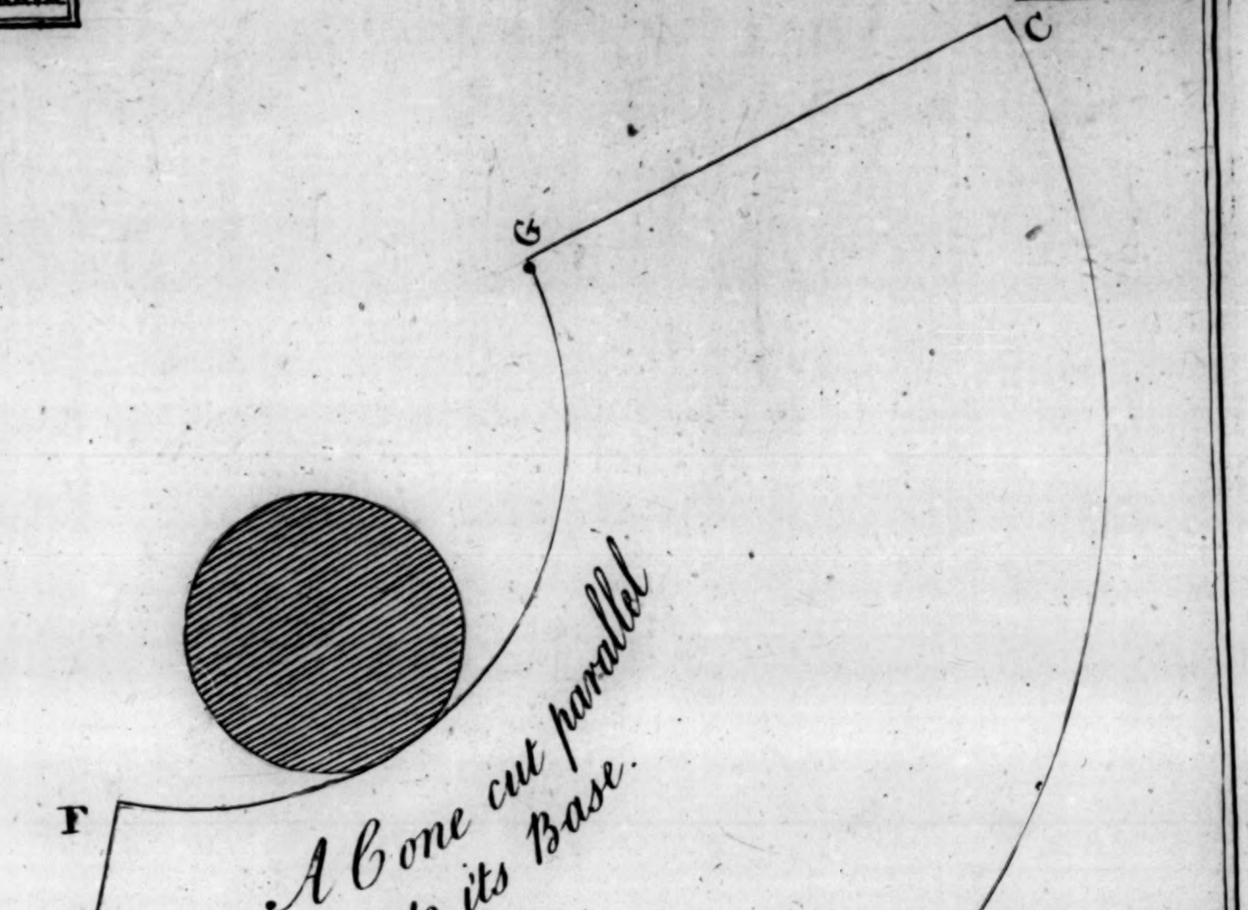





C
D
A
Cone cut
perpendicularly
through its axis
A
B
E
F
G
H
K
I






C
G
F
A Cone cut parallel to its Base.
D
A
B

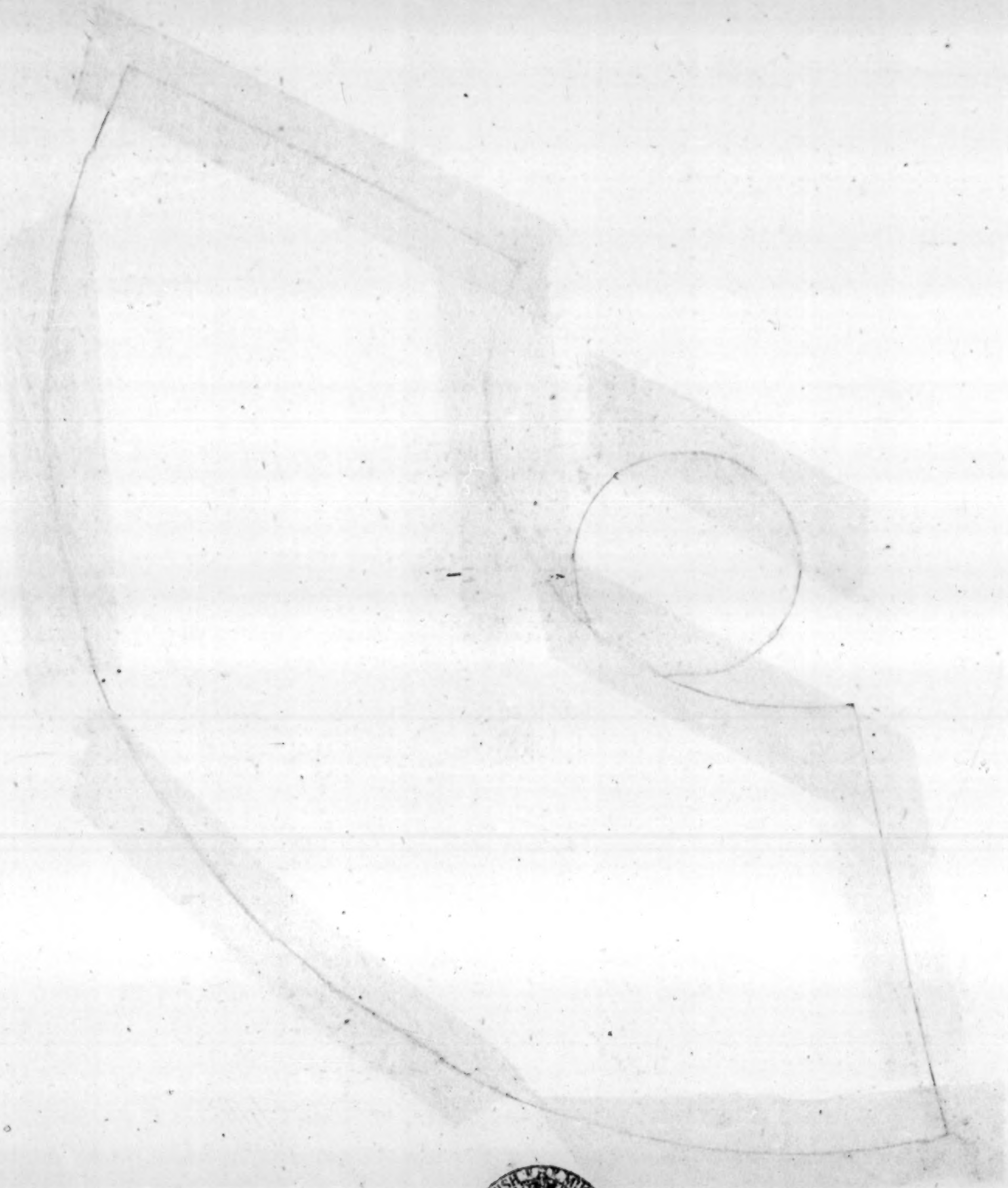
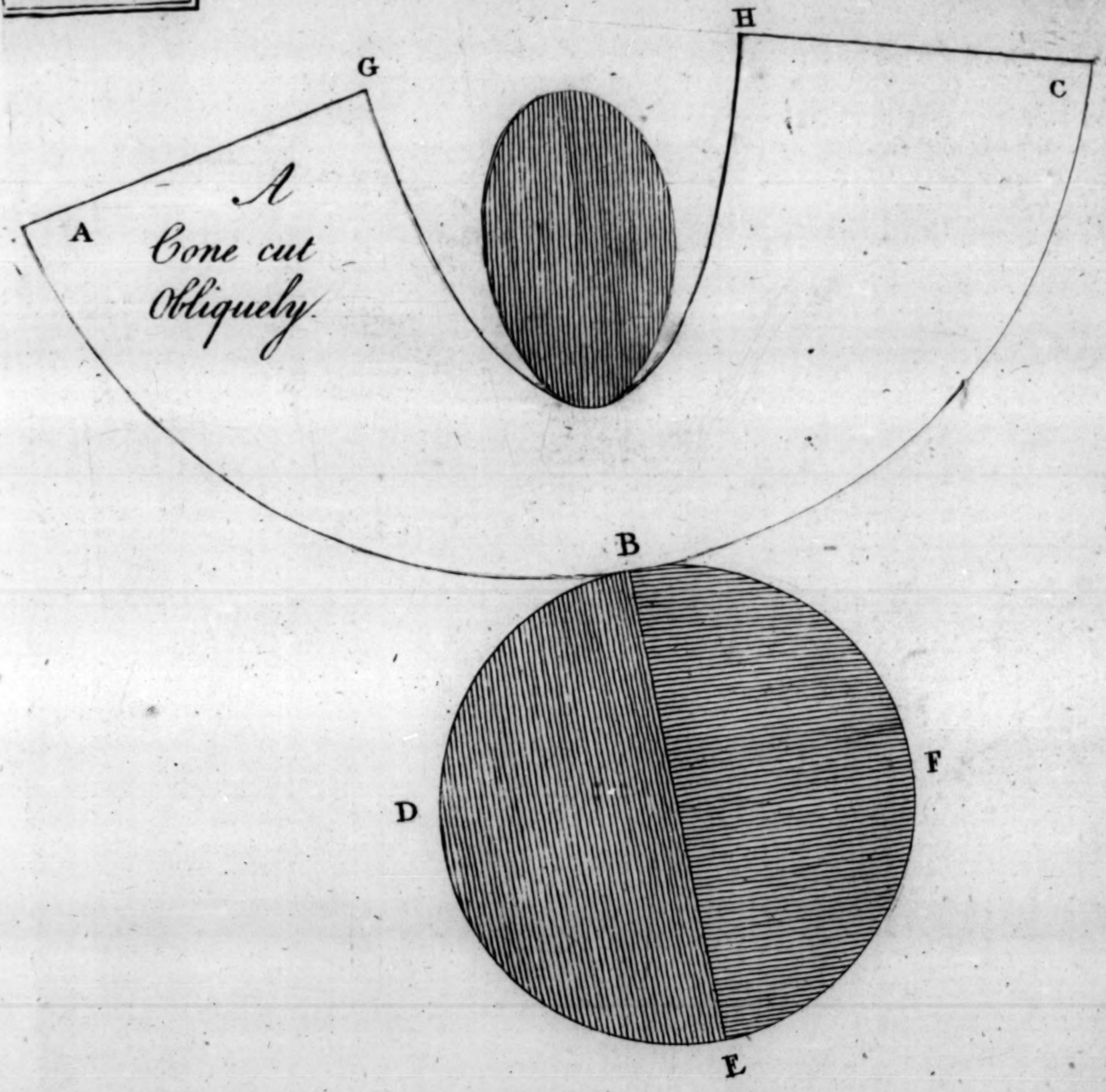


Plate XI.

Book 7.



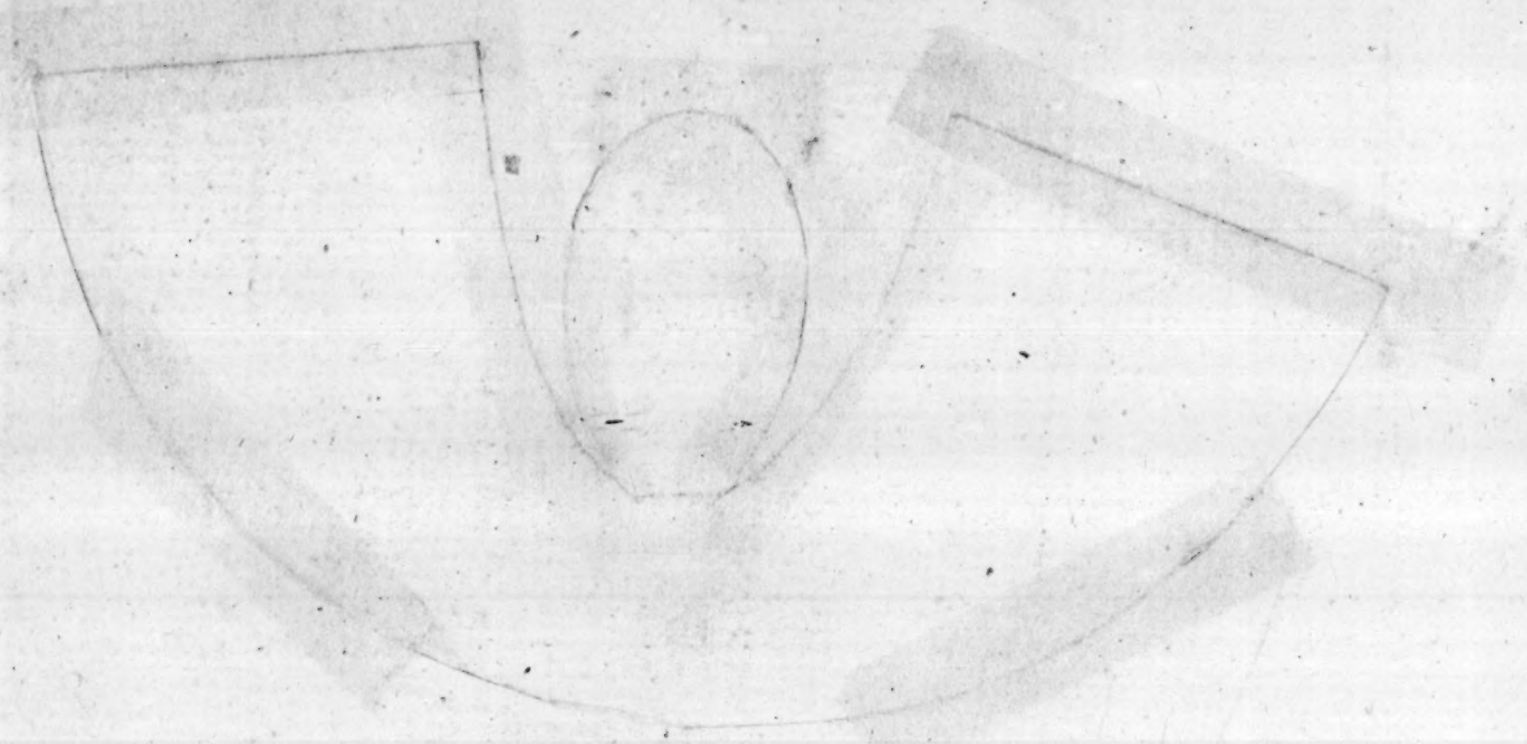


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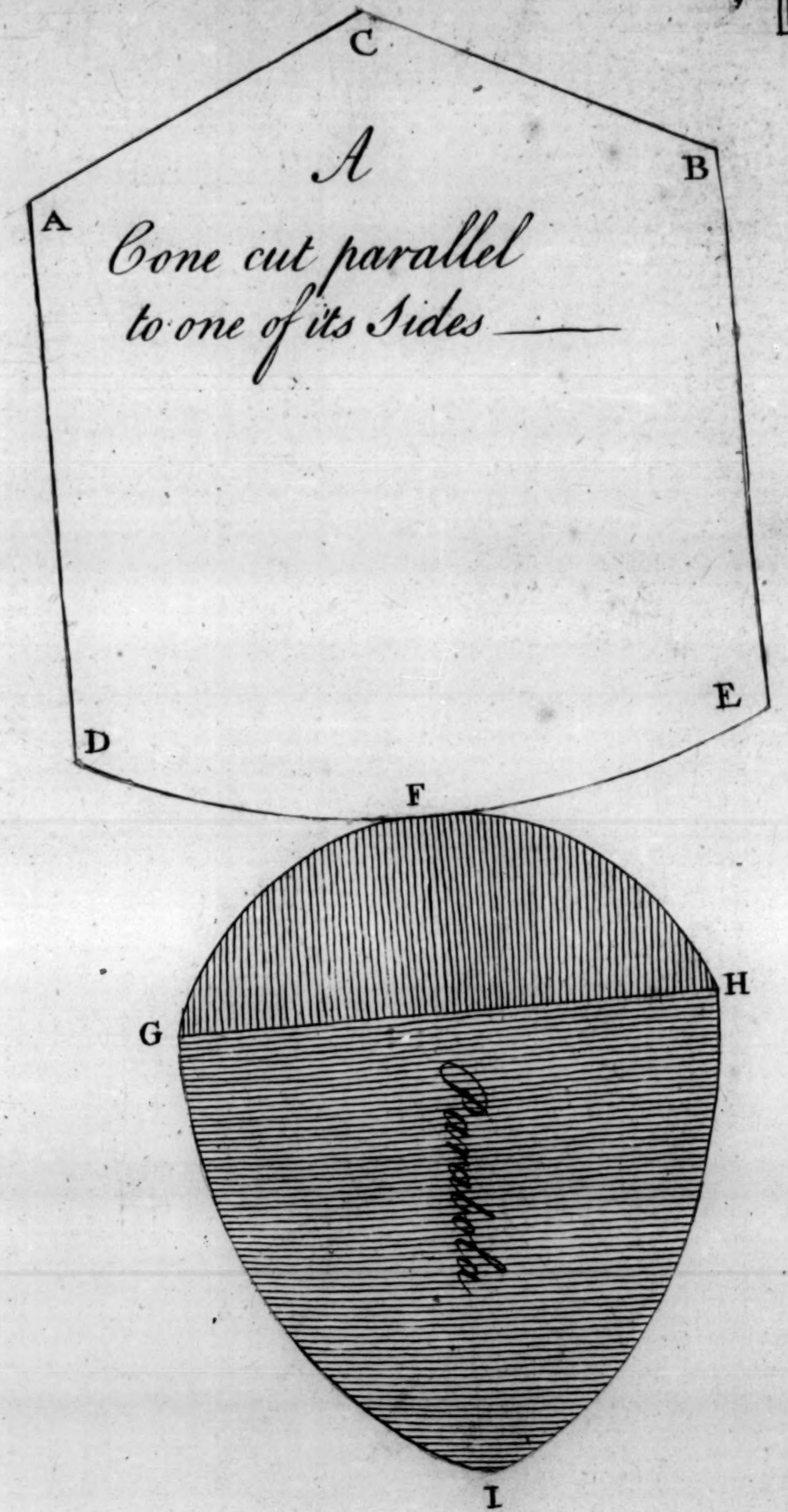
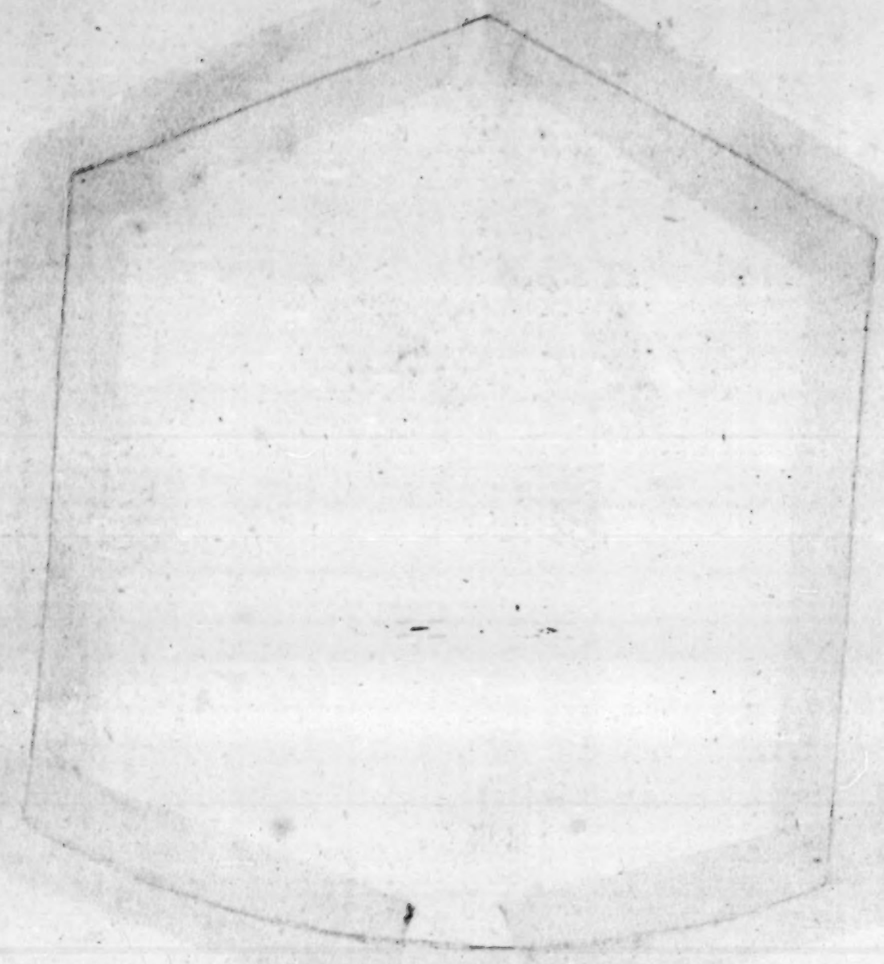
Plate XLI

Book. 7.



A Cone cut parallel to one of its Sides



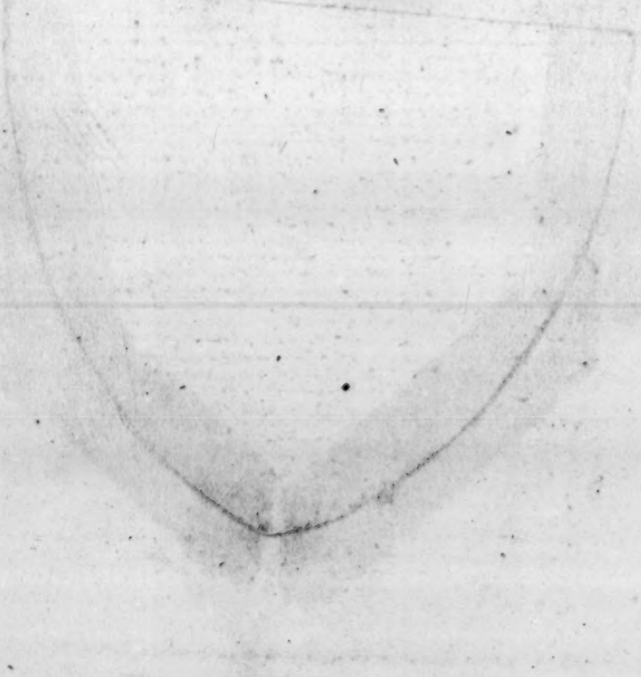


Plate XLII

Book. 7.

A
Cone cut
parallel to its Axis.